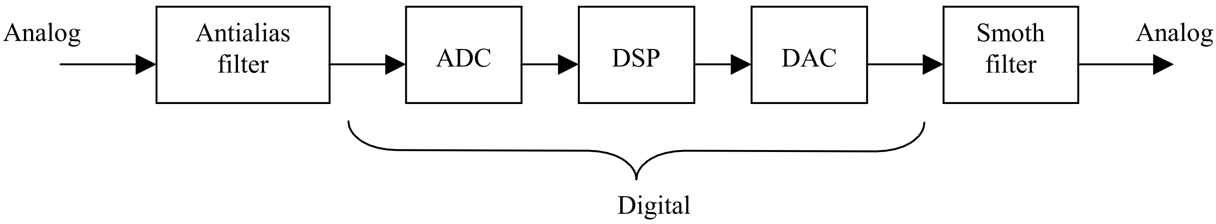



## SYGNAŁY DYSKRETNO CZASOWE:

$x_n(t)$
$x(n) = \{x(n)\} = \{\ldots, x(-1), x(0), x(1), \ldots\}$

MATHLAB:

```
>> n=[-3,-2,-1,0,1,2,3,4];x=[2,1,-1,0,1,2,3,7];
```

$x(n) = [2, 1, -1, 0, 1, 2, 3, 7];$

## TYPY SEKWENCJI:

1. IMPULS JEDNOSTKOWY (DELTA CRONECKERA)

$$\delta(n) = \begin{cases} 1\ , & n = 0 \\ 0\ , & n \neq 0 \end{cases} = \{\ldots, 0, 0, 1, 0, 0, \ldots\}$$

IMPULS JEDNOSTKOWY PRZESUNIĘTY

$$\delta(n-n_0) = \begin{cases} 1, & n = n_0 \\ 0, & n \neq n_0 \end{cases}$$

gdy $n_1 \leq n_0 \leq n_2$

WAŻNY WYNIK

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)$$

```
function [x,n] = impseq(n0,n1,n2)
%generates x(n) = delta(n-n0); n1<=n<=n2
%--------------------------------
n=[n1:n2]; x=[(n-n0)==0];
```

2. SKOK JEDNOSTKOWY

$$u(n) = \begin{cases} 1\ , & n \geq 0 \\ 0\ , & n < 0 \end{cases} = \{\ldots, 0, 0, 1, 1, 1, \ldots\}$$

SKOK JEDNOSTKOWY PRZESUNIĘTY

$$u(n-n_0) = \begin{cases} 1, & n \geq n_0 \\ 0, & n < n_0 \end{cases}$$

gdy $n_1 \leq n_0 \leq n_2$

```
function [x,n] = stepseq(n0,n1,n2)
%generates x(n) = u(n-n0); n1<=n<=n2
%-------------------------------
n=[n1:n2]; x=[(n-n0)>=0];
```

$$x(n) = a^n \quad ; \qquad \forall n : a \in R$$

```
>>n=[0:10]; x=(0.9).^n;       % .^ - mnożenie macierzy
```

3. ZESPOLONA SEKWENCJA WYKŁADNICZA

$$x(n) = e^{(\delta + j\omega_0)n} \qquad , \quad \forall n$$

```
>>n=[0:10]; x=exp((2+3j)*n);
```

4. SEKWENCJA SINUSOIDALNA

$$x(n) = \cos(\omega_0 n + \theta) \qquad , \quad \forall n$$

```
>>n=[0:10]; x=3*cos(0.1*pi*n+pi/3)+2*sin(0.5*pi*n);
```

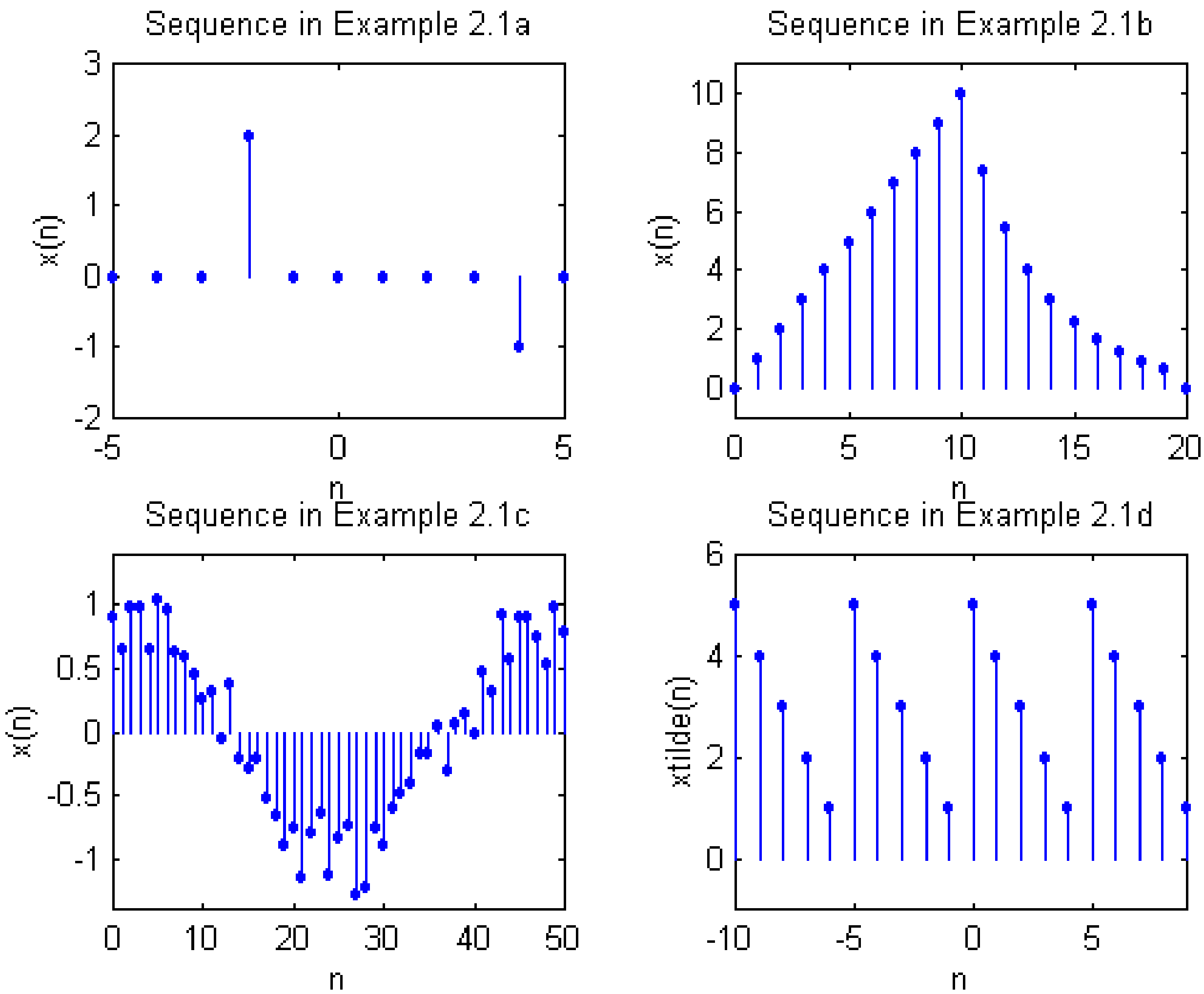
5. SEKWENCJA LOSOWA

```
>>rndn[1,N]; %wartość średnia=0  wariancja=1  N-próbek
```

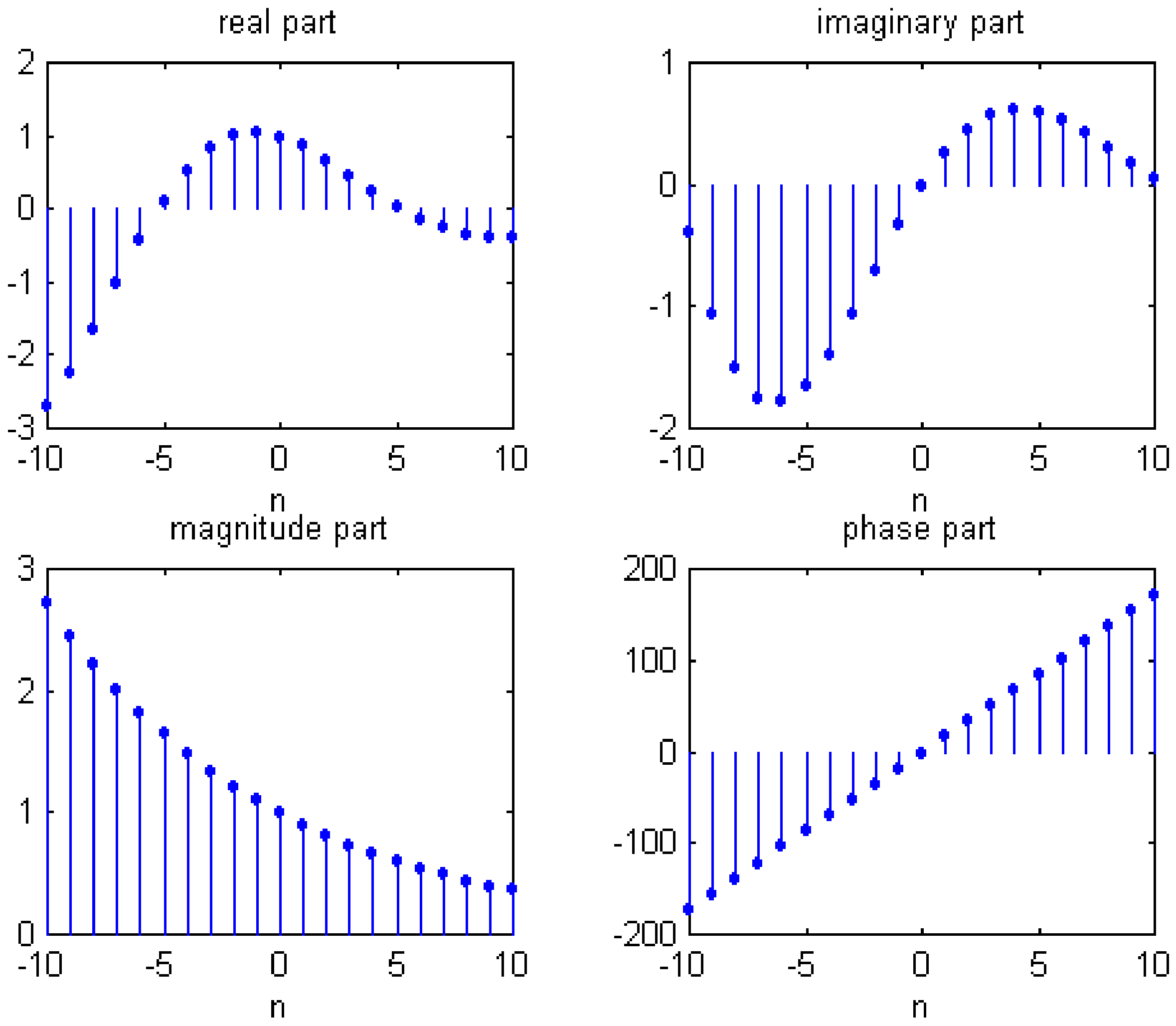
6. SEKWENCJA PERIODYCZNA $\tilde{x}(n)$

```
>>x=[1,2,3,4];
>>xtilde=x'*'ones(1,p);
>>xtilde=xtilde(:);
>>xtilde=xtilde';
```

```
figure(1); clf
% a) x(n) = 2*delta(n+2) - delta(n-4), -5<=n<=5
n = [-5:5];
x = 2*impseq(-2,-5,5)-impseq(4,-5,5);
subplot(2,2,1); stem(n,x,'.'); title('Sequence in Example 2.1a')
xlabel('n'); ylabel('x(n)'); axis([-5,5,-2,3])
%
% b) x(n) = n[u(n)-u(n-10)]+10*exp(-0.3(n-10))(u(n-10)-u(n-20)); 0<=n<=20
n = [0:20];
x1 = n.*(stepseq(0,0,20)-stepseq(10,0,20));
x2 = 10*exp(-0.3*(n-10)).*(stepseq(10,0,20)-stepseq(20,0,20));
x = x1+x2;
subplot(2,2,2);stem(n,x,'.');
title('Sequence in Example 2.1b')
xlabel('n');ylabel('x(n)');axis([0,20,-1,11])
%
% c) x(n) = cos(0.04*pi*n) + 0.2*w(n); 0<=n<=50, w(n): Gaussian (0,1)
n = [0:50];
x = cos(0.04*pi*n)+0.2*randn(size(n));
subplot(2,2,3);stem(n,x,'.');title('Sequence in Example 2.1c')
xlabel('n');ylabel('x(n)');axis([0,50,-1.4,1.4])
%
% d) x(n) = {...,5,4,3,2,1,5,4,3,2,1,...}; -10<=n<=9
n=[-10:9];
x=[5,4,3,2,1];
xtilde=x' * ones(1,4);
xtilde=(xtilde(:))';
subplot(2,2,4);stem(n,xtilde,'.');title('Sequence in Example 2.1d')
xlabel('n');ylabel('xtilde(n)');axis([-10,9,-1,6])
```


Sequence in Example 2.1a
x(n)
n
Sequence in Example 2.1b
x(n)
n
Sequence in Example 2.1c
x(n)
n
Sequence in Example 2.1d
xtilde(n)
n

```
figure(1); clf
% a) x(n) = exp((-0.1+j0.3)n), -10 <= n <= 10;
n = [-10:1:10]; alpha = -0.1+0.3j;
x = exp(alpha*n);
subplot(2,2,1); stem(n,real(x),'.');title('real part');xlabel('n')
subplot(2,2,2); stem(n,imag(x),'.');title('imaginary part');xlabel('n')
subplot(2,2,3); stem(n,abs(x),'.');title('magnitude part');xlabel('n')
subplot(2,2,4); stem(n,(180/pi)*angle(x),'.');title('phase part');xlabel('n')
```



y(n)=T[x(n)]    SYSTEM T PRZEKSZTAŁCA ciąg x(n) w y(n)

System T jest liniowy  L[ ] gdy:

$L[a_1x_1(n) + a_2x_2(n)] = a_1L[x_1(n)] + a_2L[x_2(n)], \quad \forall\ a_1, a_2, x_1(n), x_2(n)$

LINEAR TIME INVARIANT SYSTEMS  LTI

$$y(n) = L[x(n)] = L\left[\sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)\right] = \sum_{k=-\infty}^{+\infty} x(k)L[\delta(n-k)]$$

L[δ(n-k)] – jest to odpowiedź systemu na pobudzenie impulsowe h(n,k)

W SYSTEMIE LTI  h(n,k) = h(n)

$$x(n) \longrightarrow \boxed{\text{LTI [ ]}} \longrightarrow y(n) \longrightarrow \boxed{\text{SHIFTBY k}} \longrightarrow y(n-k)$$

$$x(n) \longrightarrow \boxed{\text{SHIFTBY k}} \longrightarrow x(n-k) \longrightarrow \boxed{\text{LTI [ ]}} \longrightarrow y(n-k)$$

$$y(n) = LTI\,[x(n)] = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

STABILNOŚĆ SYSTEMU

BIBO STABIITY
$|x(n)| < \infty \Rightarrow |y(n)| < \infty \ ,\forall\ x,y$

$$BIBO\ \ STABILITY \iff \sum_{-\infty}^{+\infty} |h(n)| < \infty$$

PRZYCZYNOWOŚĆ SYSTEMU (CASUALITY)

System jest przyczynowy gdy h(n)=0 , n<0;

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{N} b_m x(n-m) \ , \quad \forall n$$

$$y(n) = \sum_{m=0}^{M} b_m x(n-m) - \sum_{k=1}^{N} a_k y(n-k)$$

```
MATHLAB
y=filter(b,a,x)
b=[b0,b1,...,bM]; a=[a0,a1,...,aN];
```

FILTRY CYFROWE

IIR (ARMA)

$$y(n)=\sum_{m=0}^{M} b_m x(n-m)-\sum_{k=1}^{N} a_k y(n-k) \quad \Bigg| \quad \begin{array}{l} b=[b_0, b_1, \ldots] \\ a=[1, a_1, \ldots] \end{array}$$

FIR (MA)

$$y(n)=\sum_{m=0}^{M} b_m x(n-m) \quad \Bigg| \quad \begin{array}{l} b=[b_0, b_1, \ldots] \\ a=1 \end{array}$$

IIR (AR)

$$y(n)=x(n)-\sum_{k=1}^{N} a_k y(n-k) \quad \Bigg| \quad \begin{array}{l} b=1 \\ a=[1, a_1, a_2, \ldots] \end{array}$$

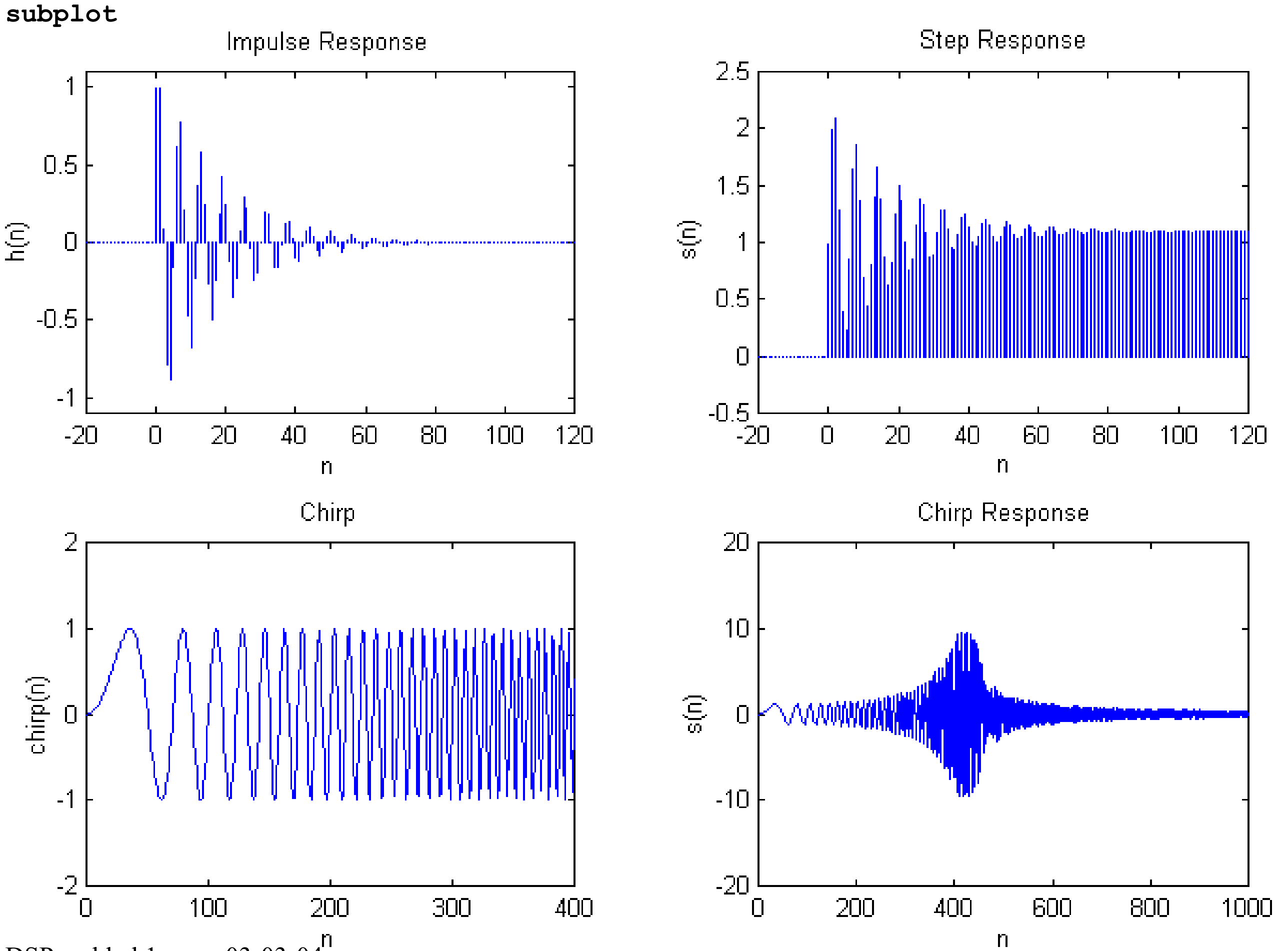
Przykład 2.9

Dane jest równanie:

$y(n) - y(n-1) + 0.9y(n-2) = x(n)$ , $\forall$ n

- Znajdź odpowiedź impulsową
- Znajdź odpowiedź na pobudzenie skokiem jednostkowym
- Czy system jest stabilny?

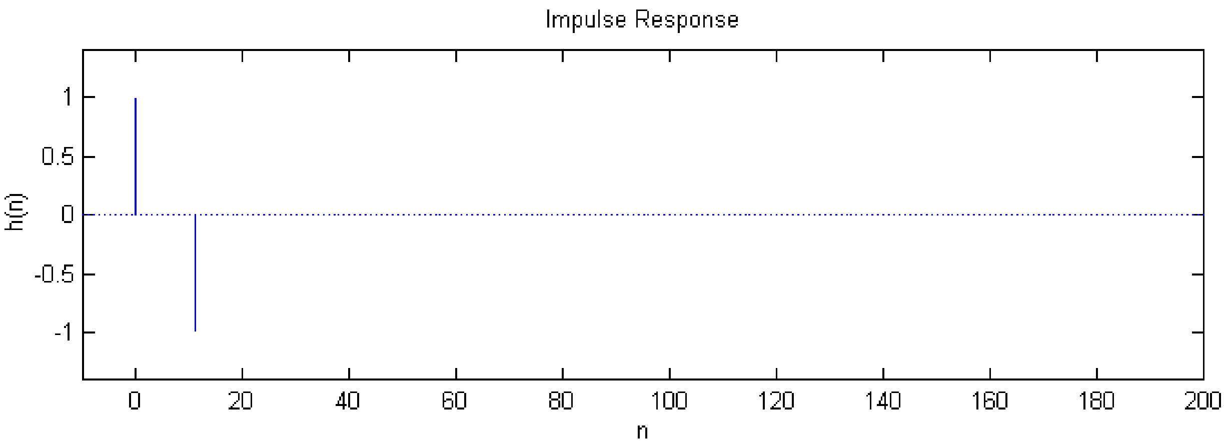
```
a=[1,-1,0.9];b=1;
% Part a)
n=[-20:120];
x=[(n-0)==0];
h=filter(b,a,x);
subplot(2,2,1);stem(n,h,'.')
axis([-20,120,-1.1,1.1])
title('Impulse Response');xlabel('n');ylabel('h(n)')
%
% Part b)
n=[-20:120];
x=[(n-0)>=0];
s=filter(b,a,x);
subplot(2,2,2);stem(n,s,'.')
axis([-20,120,-.5,2.5])
title('Step Response');xlabel('n');ylabel('s(n)')
%
% Part xx) chirp signal
n=[0:1000];
x=sin(2*pi/5000.*n.*n);
s=filter(b,a,x);
subplot(2,2,4);plot(n,s)
axis([0,1000,-20,20])
title('Chirp Response');xlabel('n');ylabel('s(n)')
subplot(2,2,3);plot(n,x)
axis([0,400,-2,2])
title('Chirp');xlabel('n');ylabel('chirp(n)')
%
%print -deps2 ex021000.eps
%
% Part c)
sum(abs(h))
z=roots(a);
magz=abs(z)
subplot
```

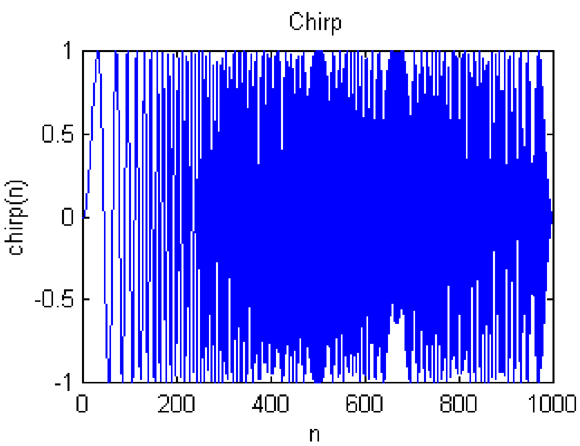


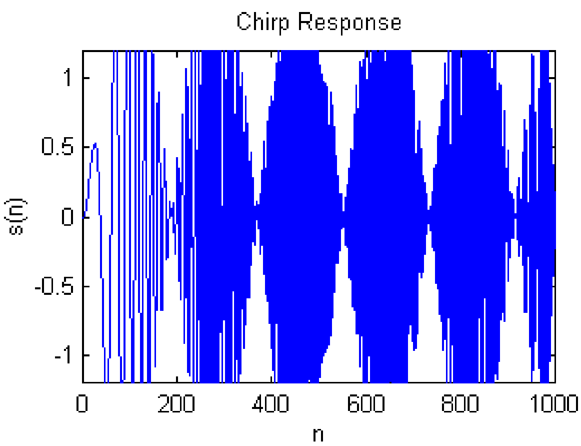
```
close;
%a=1;b=[1,zeros(1,48),0.1];        %echo  y(n)=x(n)+ax(n-k)
%a=1;b=[1,0,0,0,0,0,0,0,0,0,0,-1]; %comb filter  y(n)=x(n)-x(n-k)
a=1;b=[1,-1];                     %diferentiator  y(n)=x(n)-x(n-1)
%a=1;b=[1,1,1,1].*0.25;             %sin(x)/x low pass filter

% Part a)
n=[-10:200];
x=[(n-0)==0];
h=filter(b,a,x);
subplot(2,1,1);stem(n,h,'.')
axis([-10,200,-1.4,1.4])
title('Impulse Response');xlabel('n');ylabel('h(n)')

% Part xx) chirp signal
n=[0:1000];
x=sin(2*pi/4000.*n.*n);
s=filter(b,a,x);
subplot(2,2,4);plot(n,s)
axis([0,1000,-1.2,1.2])
title('Chirp Response');xlabel('n');ylabel('s(n)')
subplot(2,2,3);plot(n,x)
title('Chirp');xlabel('n');ylabel('chirp(n)')

%print -deps2 ex021000.eps

% Part c) BADANIE STABILNOŚCI
sum(abs(h))
z=roots(a);
magz=abs(z)
subplot
pause;

% Part a) POBUDZENIE PROSTOKĄTNE
close;
n=[-10:100];
x=5*(stepseq(0,-10,100)-stepseq(20,-10,100));
s=filter(b,a,x);
subplot(2,1,1);stem(n,x,'.');
axis([-10,100,-6,6]);
title('rectangle');xlabel('n');ylabel('rect(n)');
subplot(2,1,2);stem(n,s,'.')
axis([-10,100,-10,10])
title('rectangle response');xlabel('n');ylabel('s(n)')
pause;

% Part b) POBUDZENIE TRÓJKĄTNE
close;
n=[-10:100];
x=n.*(stepseq(0,-10,100)-stepseq(10,-10,100))-(20-n).*(stepseq(10,-10,100)-
stepseq(20,-10,100));
s=filter(b,a,x);
subplot(2,1,1);stem(n,x,'.')
axis([-10,100,-10,10])
title('triangle');xlabel('n');ylabel('trian(n)')
subplot(2,1,2);stem(n,s,'.')
axis([-10,100,-10,10])
title('triangle response');xlabel('n');ylabel('s(n)')
pause;
```

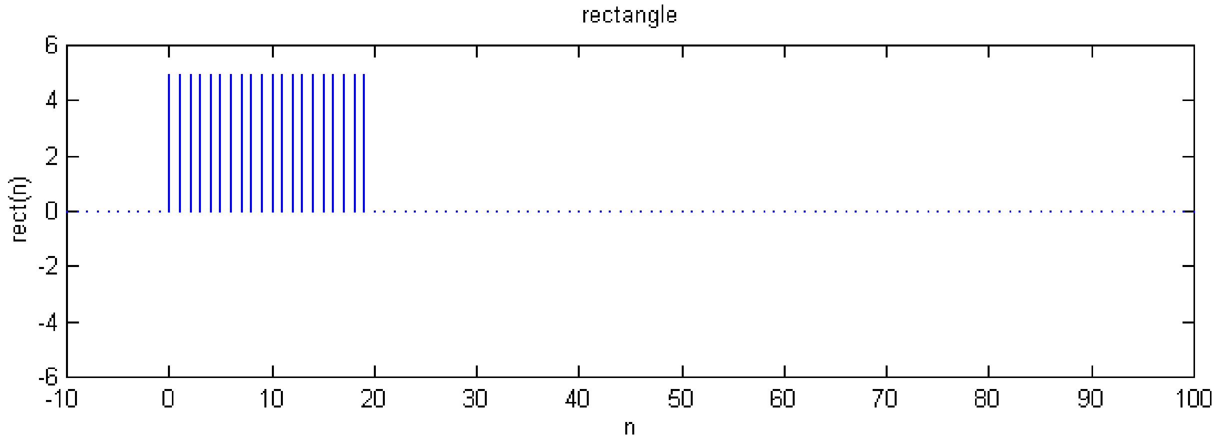
```
% Part c)  POBUDZENIE: OKRES SINUSOIDY
close;
n=[-10:300];
x=sin(pi*n/5).*(stepseq(0,-10,300)-stepseq(50,-10,300));
s=filter(b,a,x);
subplot(2,1,1);plot(n,x)
axis([-10,300,-1.2,1.2])
title('triangle');xlabel('n');ylabel('trian(n)')
subplot(2,1,2);plot(n,s)
axis([-10,300,-1.2,1.2])
title('triangle response');xlabel('n');ylabel('s(n)')
pause;

% Part x) POBUDZENIE: DUDNIENIE SYGNAŁÓW
close;
n=[0:100];
x=cos(0.1*pi*n)+0.5*cos(0.9*pi*n);
s=filter(b,a,x);
subplot(2,1,1);plot(n,x)
axis([0,100,-2,2])
title('two cosines');xlabel('n');ylabel('cos&cos(n)')
subplot(2,1,2);plot(n,s)
axis([0,100,-2,2])
title('two cosines response');xlabel('n');ylabel('s(n)')
pause;
close;
```

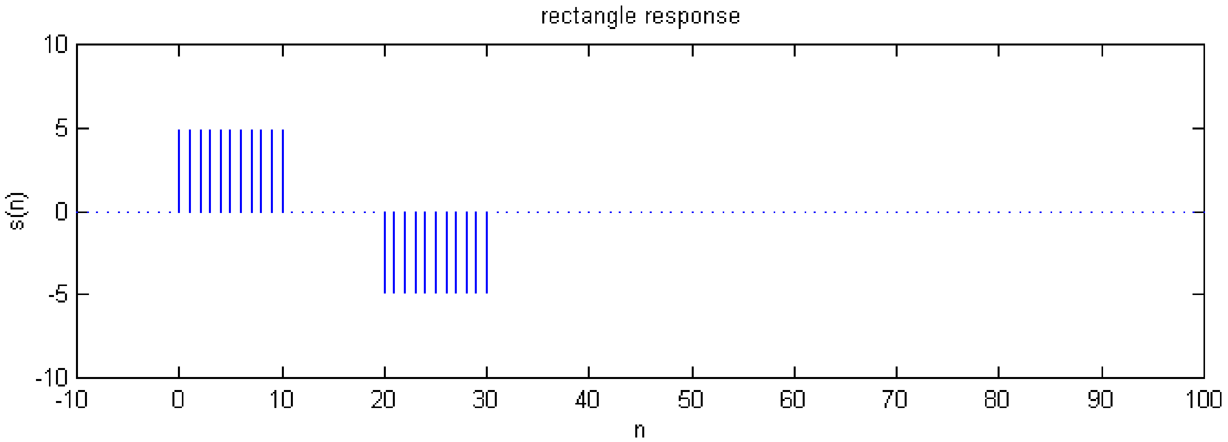
## COMB FILTER

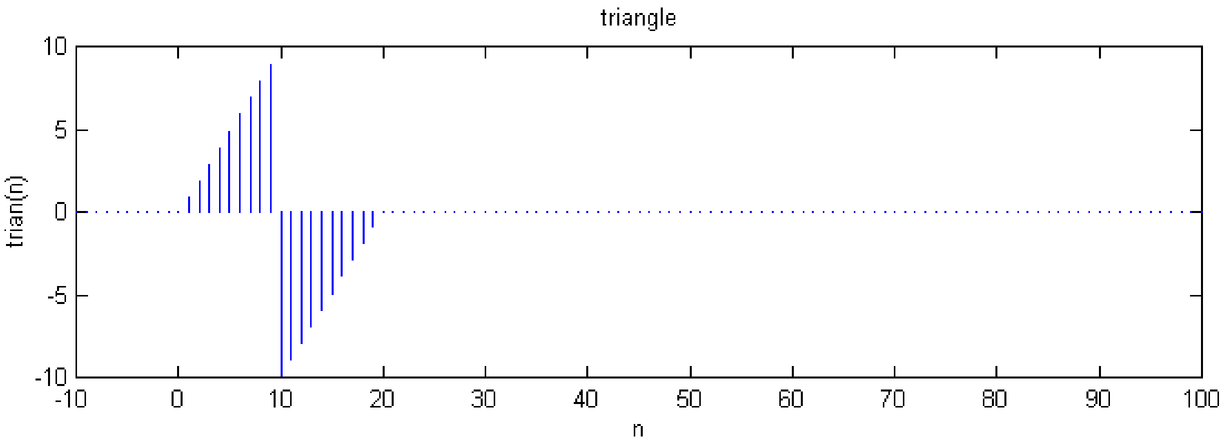


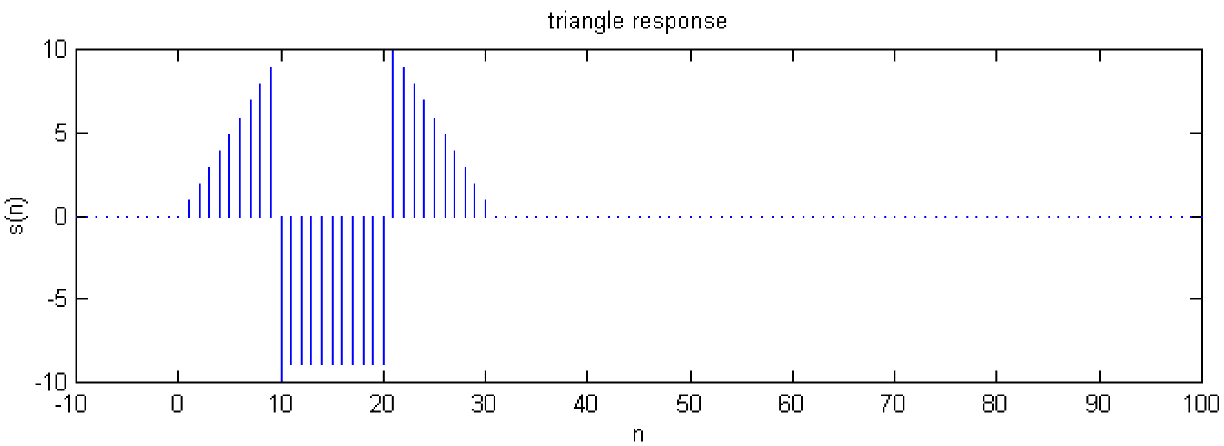



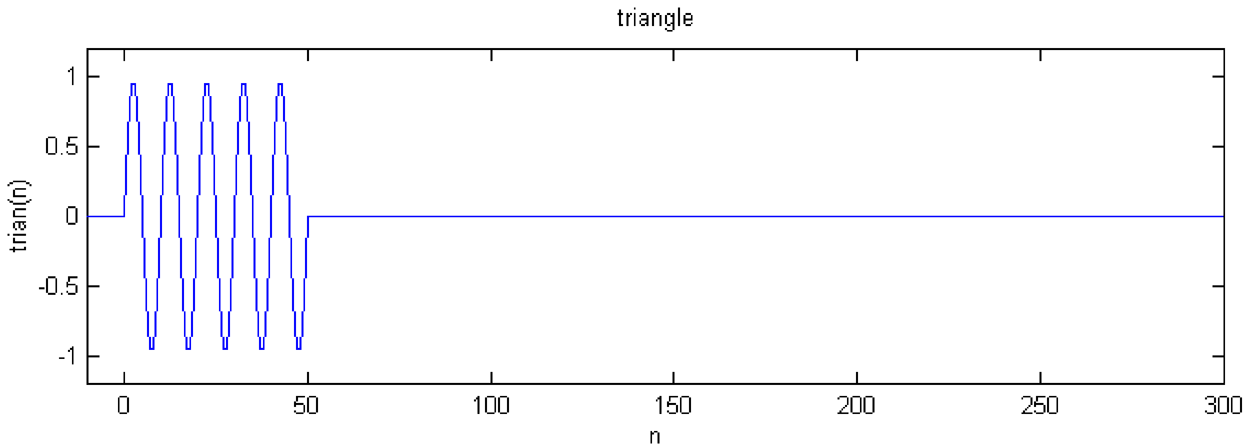



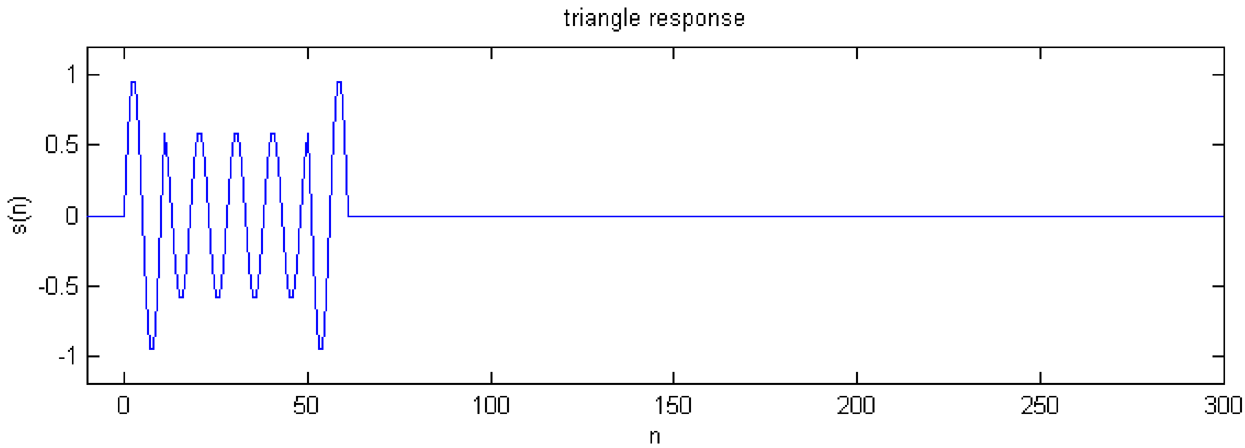

rectangle
rect(n)
6
4
2
0
-2
-4
-6
-10
0
10
20
30
40
50
60
70
80
90
100
n

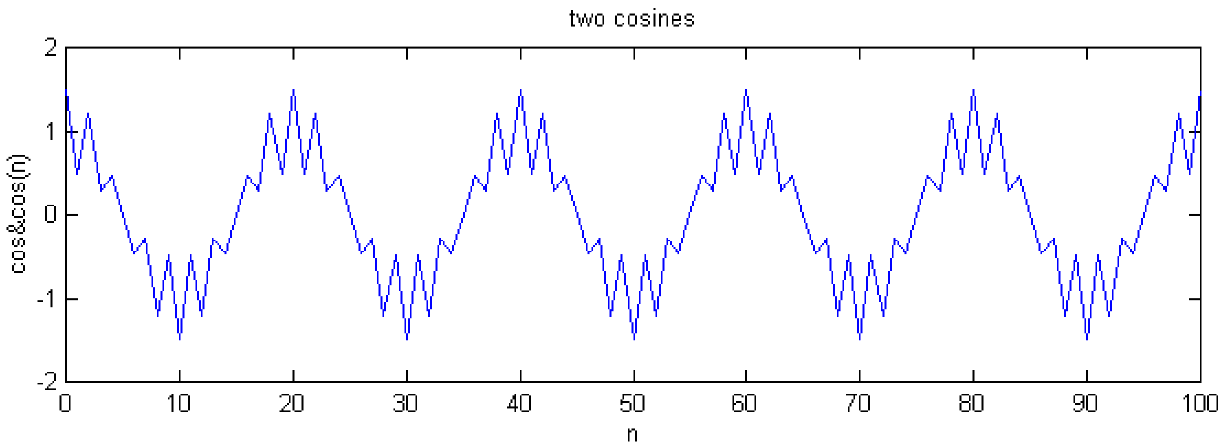

rectangle response
s(n)
10
5
0
-5
-10
-10
0
10
20
30
40
50
60
70
80
90
100
n

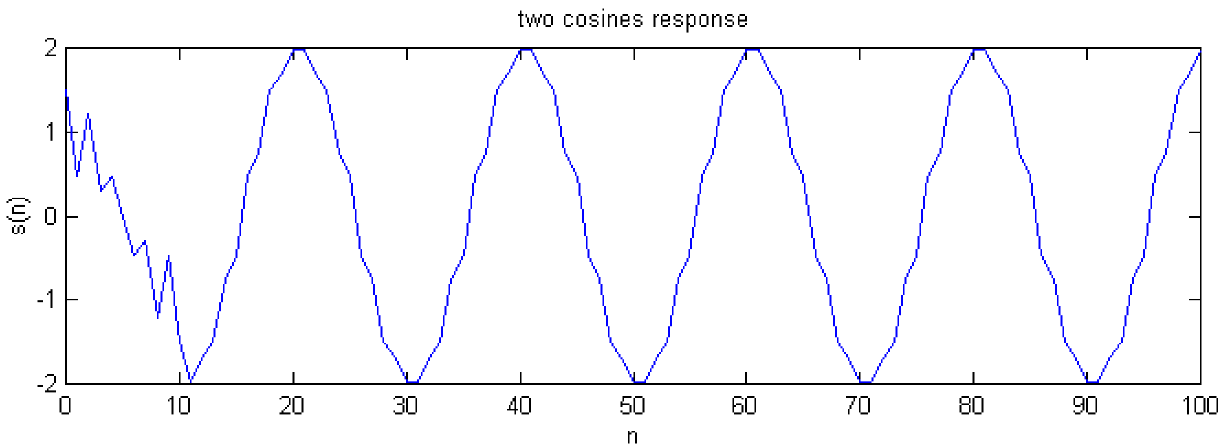

triangle
trian(n)
10
5
0
-5
-10
-10
0
10
20
30
40
50
60
70
80
90
100
n


triangle response
s(n)
10
5
0
-5
-10
-10
0
10
20
30
40
50
60
70
80
90
100
n

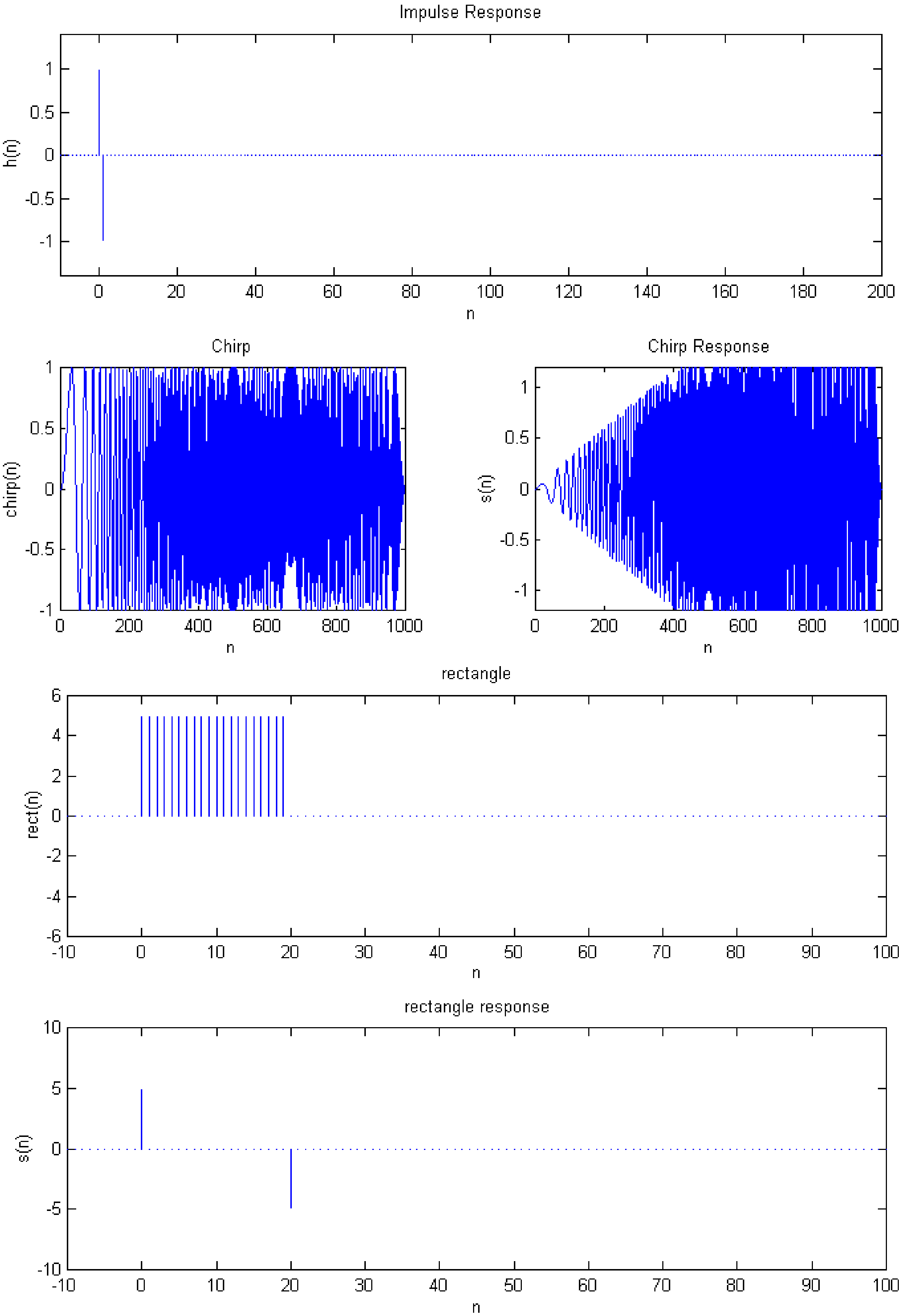

triangle
trian(n)
1
0.5
0
-0.5
-1
0
50
100
150
200
250
300
n

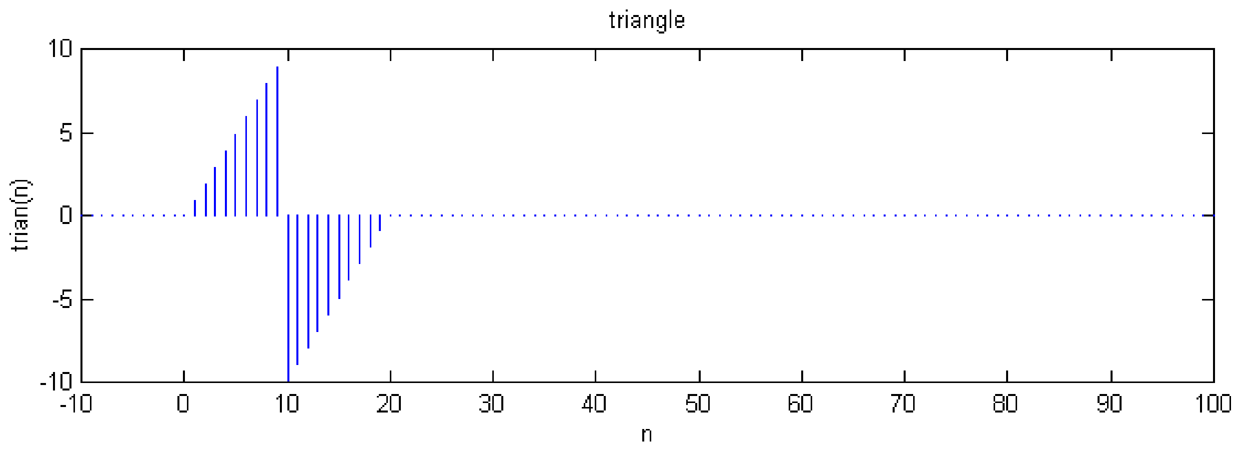

triangle response
s(n)
1
0.5
0
-0.5
-1
0
50
100
150
200
250
300
n

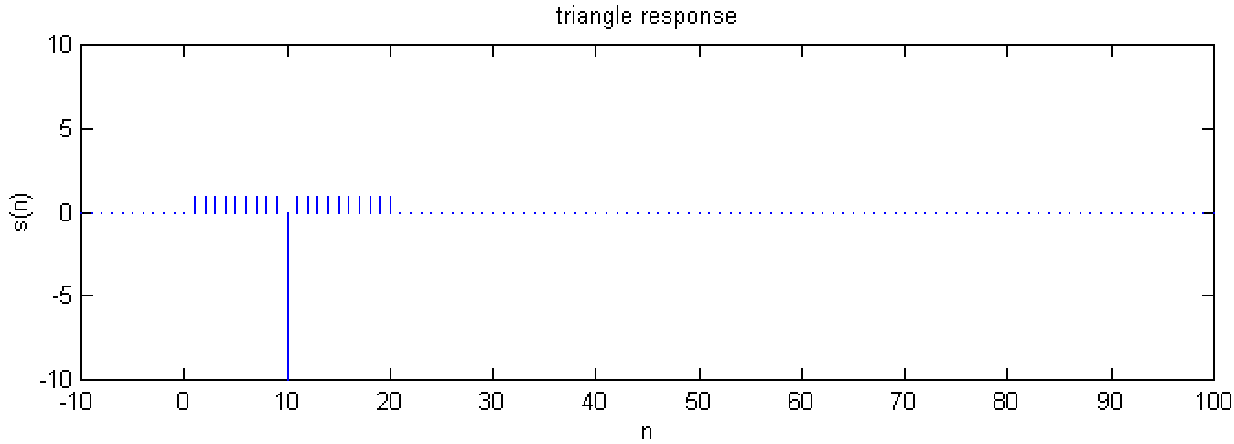

two cosines
cos&cos(n)
2
1
0
-1
-2
0
10
20
30
40
50
60
70
80
90
100
n

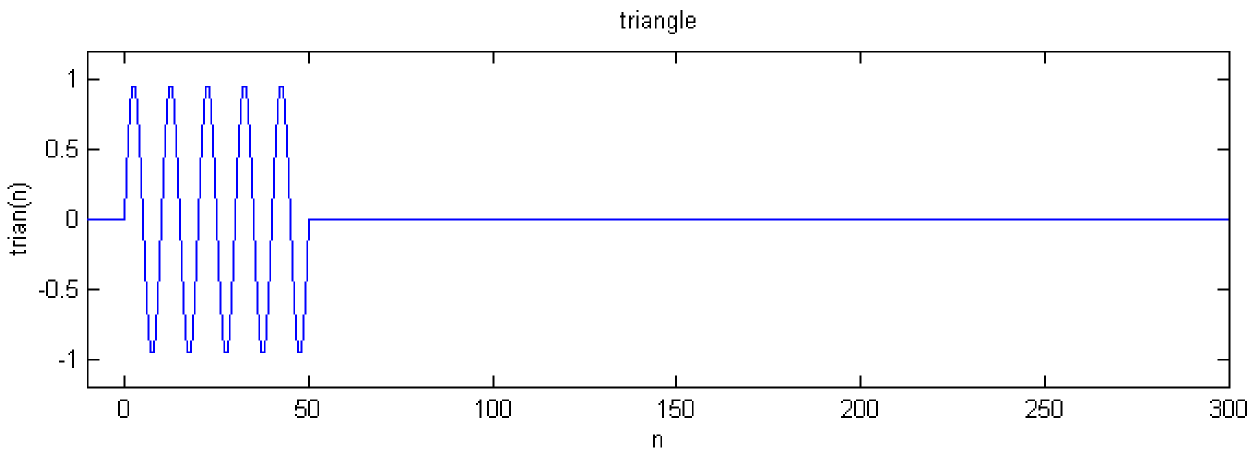

two cosines response
s(n)
2
1
0
-1
-2
0
10
20
30
40
50
60
70
80
90
100
n

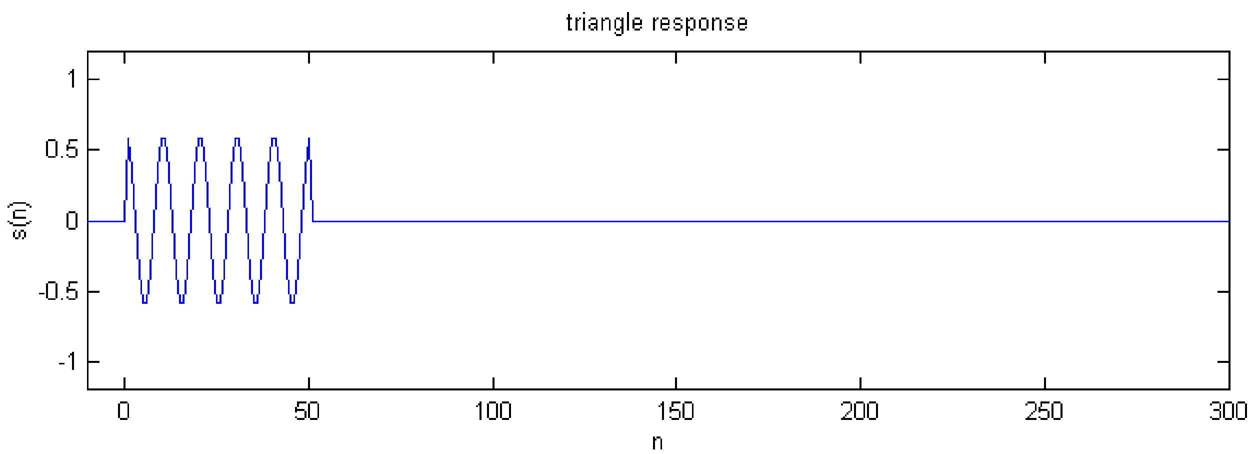
# DIFERENTIATOR

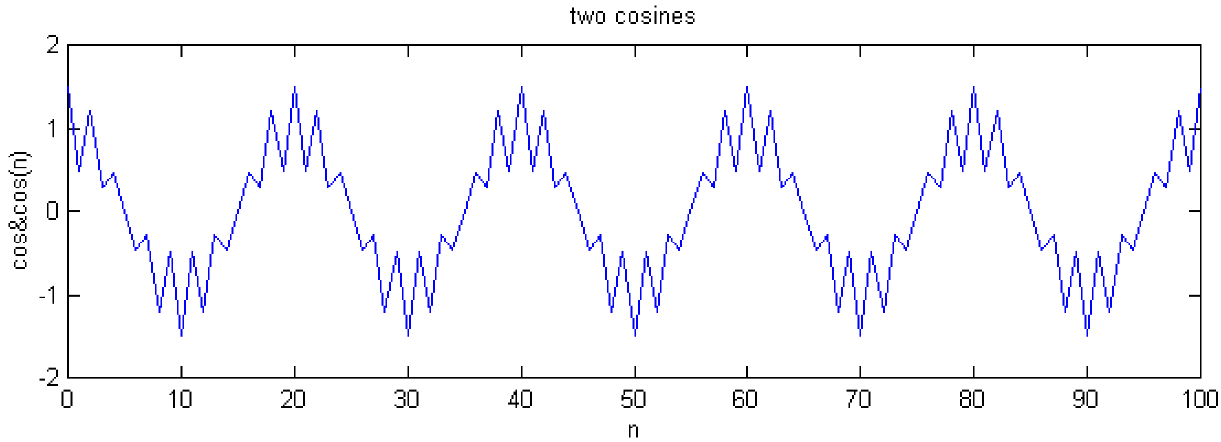


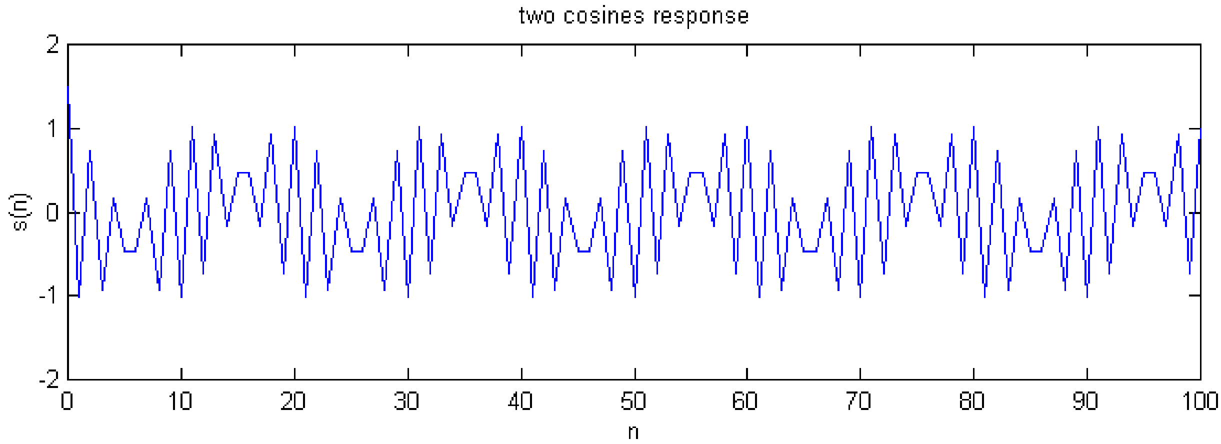

triangle
trian(n)
n
10
5
0
-5
-10
-10 0 10 20 30 40 50 60 70 80 90 100


triangle response
s(n)
n
10
5
0
-5
-10
-10 0 10 20 30 40 50 60 70 80 90 100

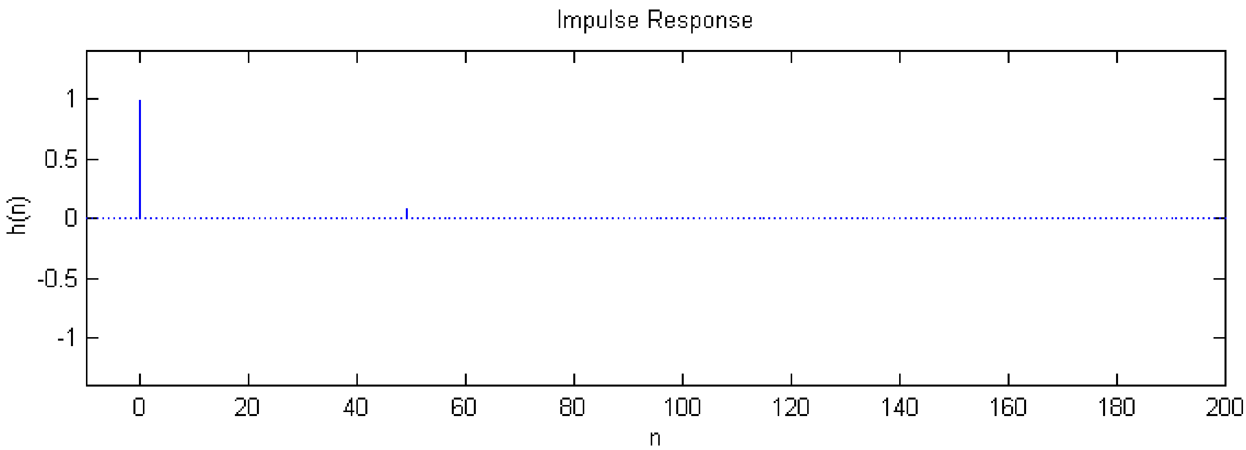

triangle
trian(n)
n
1
0.5
0
-0.5
-1
0 50 100 150 200 250 300

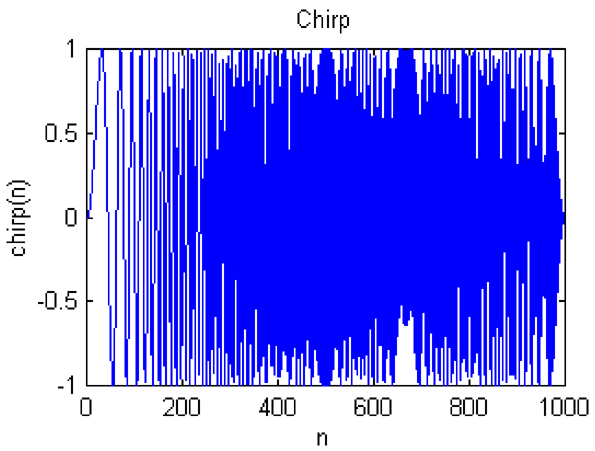

triangle response
s(n)
n
1
0.5
0
-0.5
-1
0 50 100 150 200 250 300

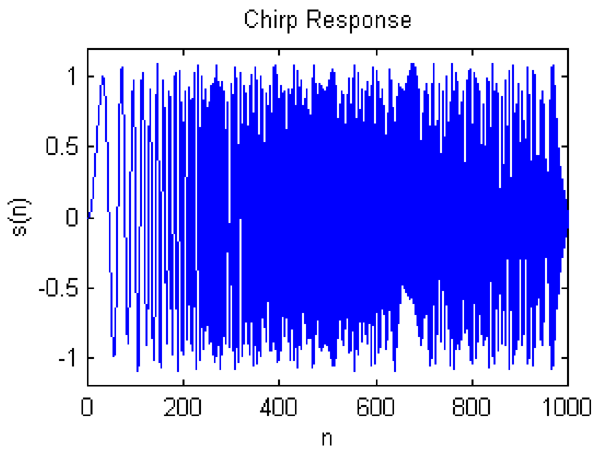

two cosines
cos&cos(n)
n

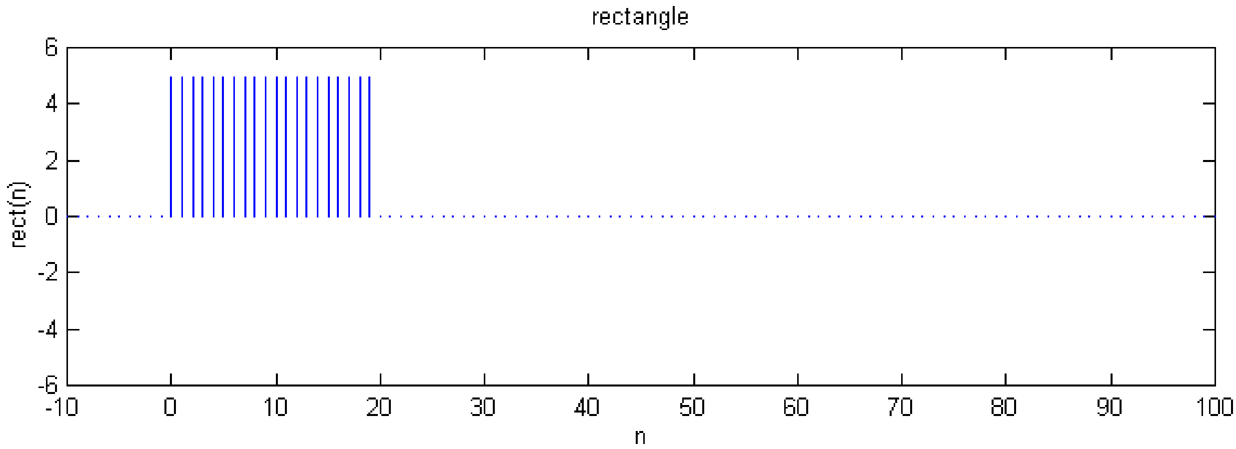

two cosines response
s(n)
n

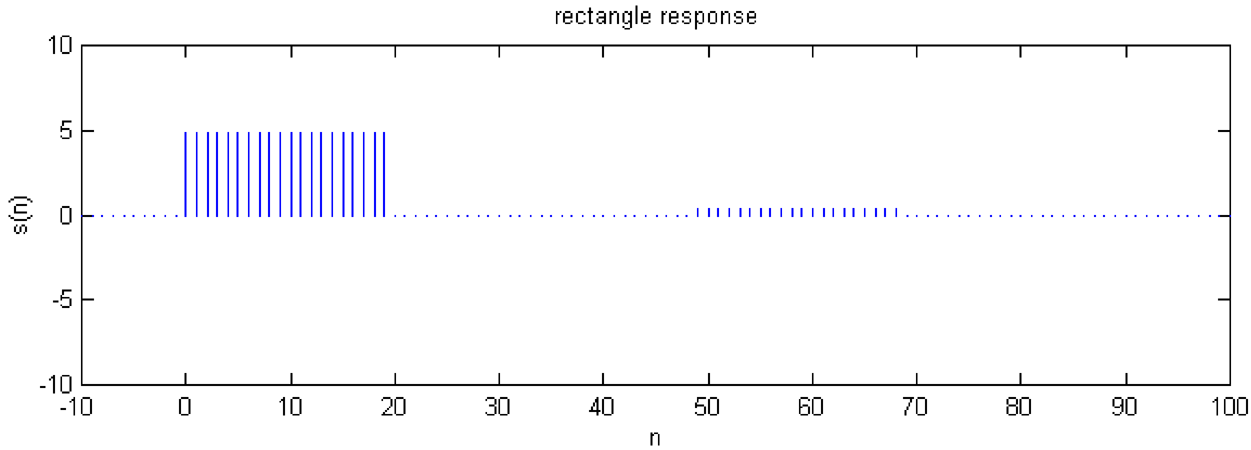
# ECHO

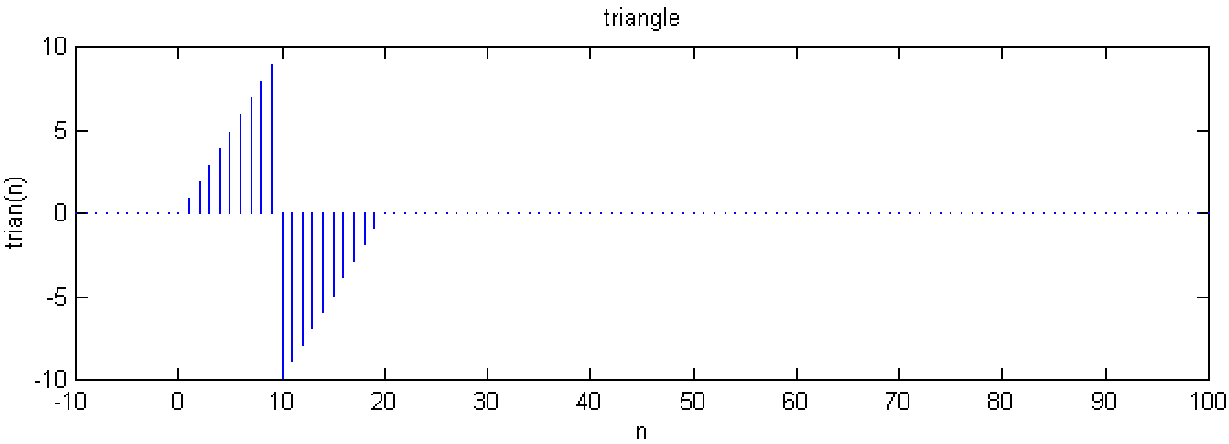

Impulse Response
h(n)
n

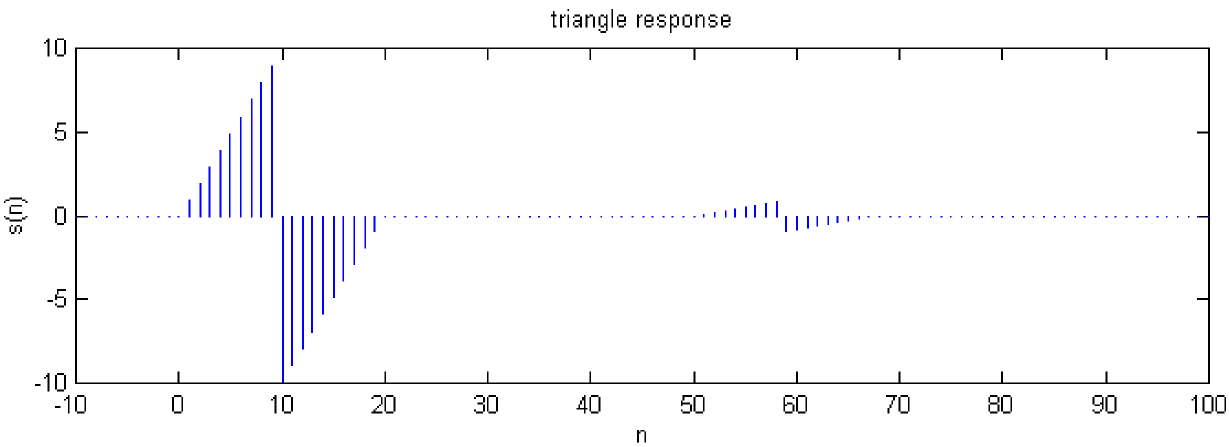

Chirp
chirp(n)
n

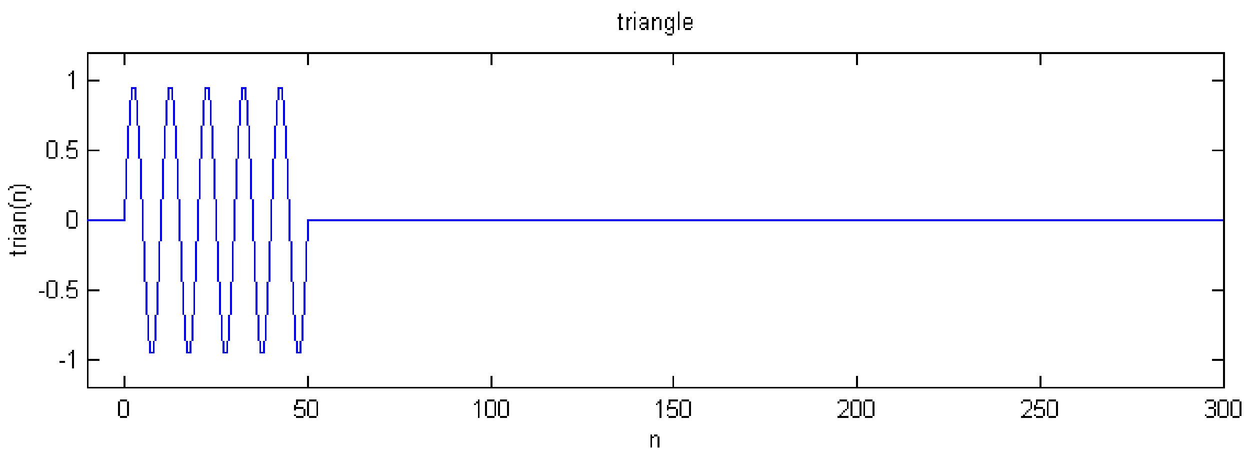

Chirp Response
s(n)
n

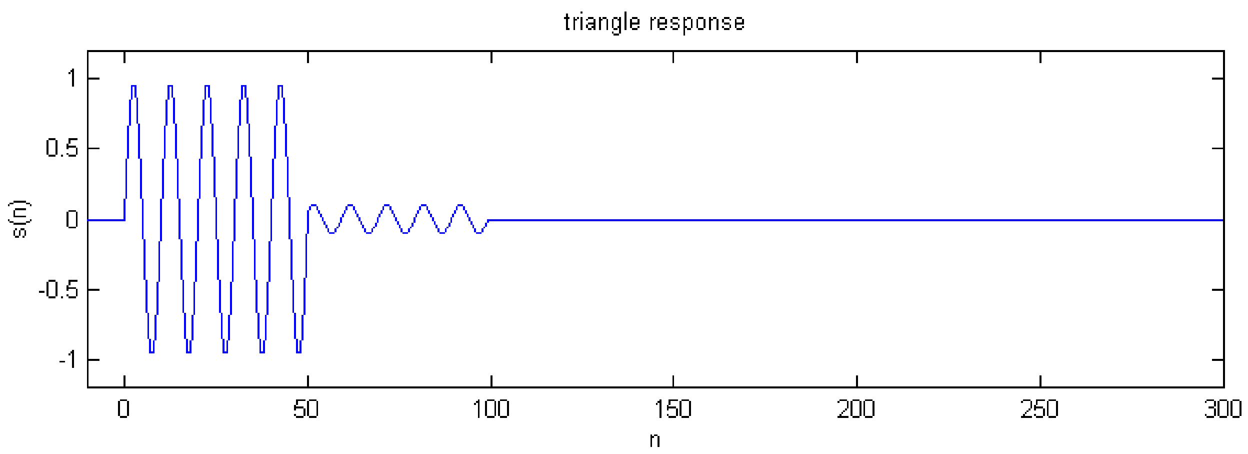

rectangle
rect(n)
n

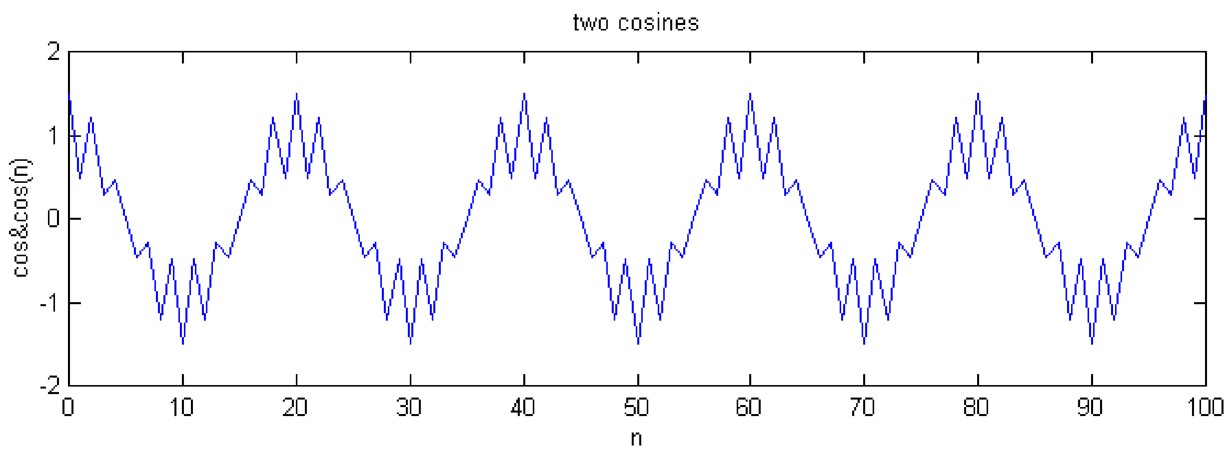

rectangle response
s(n)
n

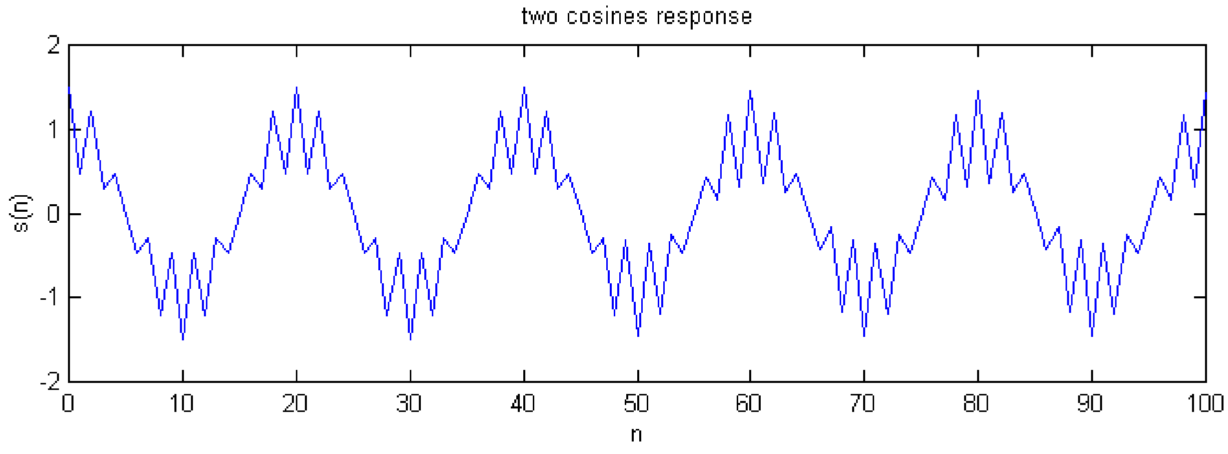

triangle
trian(n)
n


triangle response
s(n)
n


triangle
trian(n)
1
0.5
0
-0.5
-1
0
50
100
150
200
250
300
n


triangle response
s(n)
1
0.5
0
-0.5
-1
0
50
100
150
200
250
300
n


two cosines
cos&cos(n)
2
1
0
-1
-2
0
10
20
30
40
50
60
70
80
90
100
n


two cosines response
s(n)
2
1
0
-1
-2
0
10
20
30
40
50
60
70
80
90
100
n