Filtry IIR

Transmitancja filtru

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_M z^{-M}} =$$

$$= C \frac{(z - z_0)(z - z_1)\ldots(z - z_N)}{(z - p_0)(z - p_1)\ldots(z - p_M)} =$$

zera

bieguny

$$= C \prod_{k=1}^{N/2} \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = H_1(z).H_2(z).\ldots =$$

Realizacja szeregowa (kaskadowa)

Sekcja kwadratowa

$$= \frac{a_n}{b_n} + \sum_{k=1}^{N} \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Realizacja równoległa

Postać równania różnicowego:

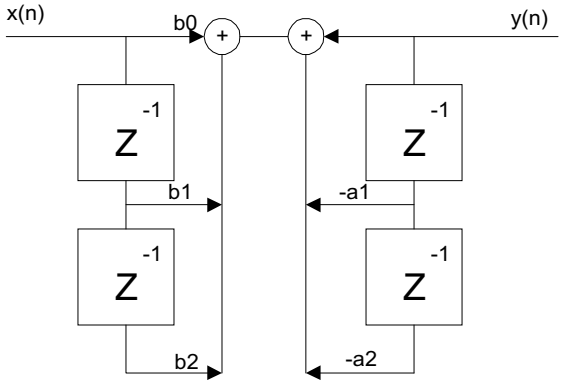
$$y(n) = \sum_{k=0}^{N} b_n x(n-k) - \sum_{k=1}^{M} a_n y(n-k)$$

Sekcja kwadratowa (biquad section)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{N(z)}{D(z)}$$

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) - a_1 y(n-1) - a_2 y(n-2)$$

Postać bezpośrednia filtru



Postać kanoniczna sekcji kwadratowej

$$\frac{Y(z)}{X(z)} = \frac{N(z)}{D(z)}$$

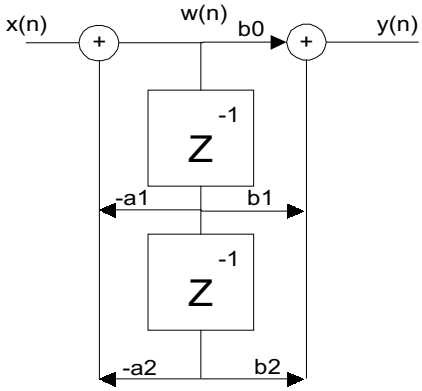
$$Y(z) = N(z)\frac{X(z)}{D(z)}$$

$$Y(z) = N(z)W(n)$$

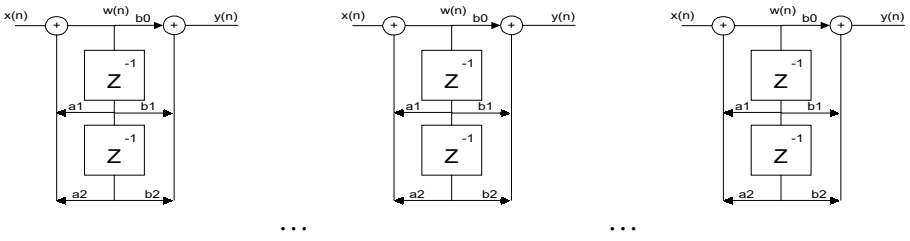
$$W(n) = \frac{X(z)}{D(z)} = \frac{X(z)}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2)$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2)$$



Kaskadowe łączenie sekcji kwadratowych



Realizacja filtru IIR w języku C (operacje typu *float***)**

Sekcja kwadratowa użyta w programie (biquad section)

$$H(z) = \frac{Y(z)}{X(z)} = C\frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{N(z)}{D(z)}$$

```
% iir_chec.m - badanie filtrów IIR
clf;
%b=[ 1 -1.62 1];
%a=[ 1  -1.52  0.88];  % notch filter
%b=[ 0 0.31 0];        % lowpass
%a=[ 1  -1.03  0.35];
%b=[ 0.09 -0.018 0.09]; % RC filter
%a=[ 1  -1.005  0.37];
%b=[ 1 3 3 1].*0.143;           % Buterwotrh
%a=[ 1  -0.18  0.34 -0.02];
%b=[ 1 0 -1 ].*0.137;           % Buterwotrh band
%a=[ 1  -1.24  0.73];
%b=[ 1 0 -1 ].*0.14;            % Buterwotrh low
%a=[ 1  0.59  0.76];
%b=[ 1 3 3 1 ].*0.14;           % Buterwotrh low
%a=[ 1 -0.18 0.34  -0.02];
%b=[ 1 -1 ];           % example 3.13 high pass
%a=[ 1 +0.5];
%b=[ 1 1 ];            % example 3.13 high pass
%a=[ 1 -0.7071];
%b=[ 1 -1.62 1];           % fir
%a=[ 1 0 0 ];
%b=[ 1 -1.73 1];
%a=[ 1  -1.56  0.81];     % notch filter kollokwium 2
%b=[ 1 -1.618 1];
%a=[ 1 ];                 % fir filter kollokwium 1

%b= [1/j, -0.97/j,  1/j]; % egz popr
%a= [1,-0.94, 0.73];
%b= [1,     1];
%a= [1, 0.18 ];
%b = [0.0181,  0.0543, 0.0543,  0.0181];
%a = [1.0000, -1.7600, 1.1829, -0.2781];
fi=pi/4;
radius=0.9;
a=poly([radius*exp(j*fi), radius*exp(-j*fi)]) %zera i bieguny
b=poly([exp(j*fi), exp(-j*fi)])

    hold off;               % <<<< MATLAB 5.0 !!!!!
    z=cplxgrid(30);
    % disp(z);
    w=polyval(b,z).*(polyval(a,z).^-1);
    cplxmap(z, w );
    % colormap(hsv);
    grid on;
    pause;
    surf(real(z),imag(z), abs(w ));
    hold off;
    grid on;
    pause;
    grid off;
    contour(real(z),imag(z), abs(w ));
    hold off;
    pcolor(real(z),imag(z), abs(w ));
    shading interp ;
    pause;
zplane(b,a);
pause;
[H,r]=freqz(b,a,1024);
plot(r*2/(2*pi),abs(H));
grid on;
pause;
plot(r*2/(2*pi),angle(H));
%plot(r*2/(2*pi),20*log10(abs(H)));
grid on;
```

```
pause;
%clg;
N=12;
t=[1 zeros(1,N-1)]
y=filter(b,a,t);
stem(y,'.');
disp('imp');
disp(y);
grid on;
axis([0 N+1  min(y)*1.1 max(y)*1.1]);
disp('roots(b)');
roots(b);
disp('roots(a)');
roots(a);
disp(' b: abs angle');
abs(roots(b)')
angle(roots(b)').*180/pi
disp(' a: abs angle');
abs(roots(a)')
angle(roots(a)').*180/pi
```

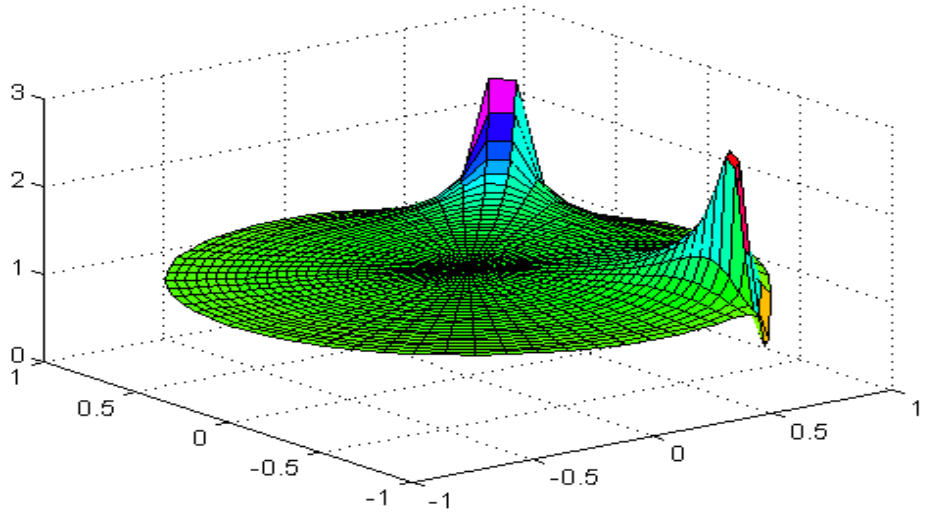
```
% iir_desi.m  : filtr iir forma bezp., kaskada
cascade=1;
direction= 'down';
N=100;
%[b,a]    = ellip(5,0.3,40.2,[0.2],'high'); %N-1, dB, dB, w <<<highpass
%[z,p,k]  = ellip(5,0.3,40.2,[0.2],'high');

%[b,a]    = ellip(5,0.3,40.2,[0.2,0.3],'stop' );
%[z,p,k]  = ellip(5,0.3,40.2,[0.2,0.3],'stop' );

%[b,a]    = butter(5,[0.2],'high'); %N-1,  w  <<<highpass
%[z,p,k]  = butter(5,[0.2],'high');

[b,a]    = maxflat(5,5,[0.2] );  % NB-1, NA-1, w <<< lowpass
z= roots(b);
p= roots(a);
k=1;

%[b,a]    = cheby2(5,100,[0.2], 'high'); %N-1, dB, w  <<<highpass
%[z,p,k]  = cheby2(5,100,[0.2], 'high');

%[b,a]    = cheby1(5,0.1,[0.2], 'high'); %N-1, dB, w  <<<highpass
%[z,p,k]  = cheby1(5,0.1,[0.2], 'high');


[H,w]=freqz(b,a,1024);
subplot(2,2,1);
plot(w*2/(2*pi),20*log10(abs(H)));
grid on;
subplot(2,2,3);
plot(w*2/(2*pi),(angle(H)));
grid on;
subplot(2,2,2);
zplane(b,a);
grid on;
subplot(2,2,4);
impz(b,a,N,'.');
grid on;
disp( b);
disp( a);
pause;
clf;
x=[1 zeros(1,N-1)]
y=filter(b,a,x);
stem(y,'.');
grid on;
```

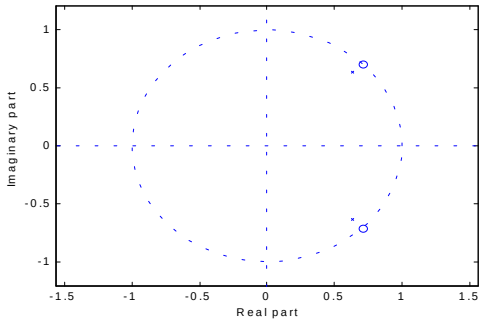
```
pause;


%  chirp signal

n=[0:1000];
x=sin(2*pi/5000.*n.*n);  %chirp

%x=wavread('c:\mat\car.wav');
y=filter(b,a,x);
pause
sound(x);
pause;
sound(y);
subplot(2,2,4);plot(n,y)
title('Chirp Response');xlabel('n');ylabel('y(n)')
axis([0,1000,-1.2,1.2])
subplot(2,2,3);plot(n,x)
title('Chirp');xlabel('n');ylabel('chirp(n)')
axis([0,1000,-1.2,1.2])
pause;
pause

clf;
    sos = zp2sos(z,p,k,direction);
    disp(sos);
    np = 256; m = size(sos,1);
    disp(m);
    h = []; H = zeros(np,m);
    [H(:,1),F] = freqz(sos(1,1:3),sos(1,4:6),np,2);
    x=ones(size(F));
    plot3(x,F,20*log10(abs(H(:,1))),'b');
    grid on;
    hold on;
    for i=2:m,
      if cascade,
         H(:,i) = H(:,i-1).*freqz(sos(i,1:3),sos(i,4:6),np,2);
         plot3(x.*i, F,20*log10(abs(H(:,i))),col(i));
      else
          H(:,i) = freqz(sos(i,1:3),sos(i,4:6),np,2);
          plot3(x.*i, F,20*log10(abs(H(:,i))),col(i));

      end
    end
hold off;

pause;

% iir w postaci programu ---------------------------------------------
close;

n=[0:1000];
%x=randn(1,1001);        %szum
%x=sin(2*pi/5000.*n.*n);  %chirp
x=[(n==20)];           %impulse
%t=0:0.0001:1;
%x=chirp(t,0,1,10000);

y=zeros(1,length(x));
NN=m;
hist=zeros(NN,2);
coef = sos;
for j=1:length(x),
    out = x(j) ;

    for i = 1 : NN,
        history1 = hist(i,1);           %/* history values */
        history2 = hist(i,2);

        new_hist = out      * coef(i,4) - history1 * coef(i,5) - history2 * coef(i,6) ;  % poles
```

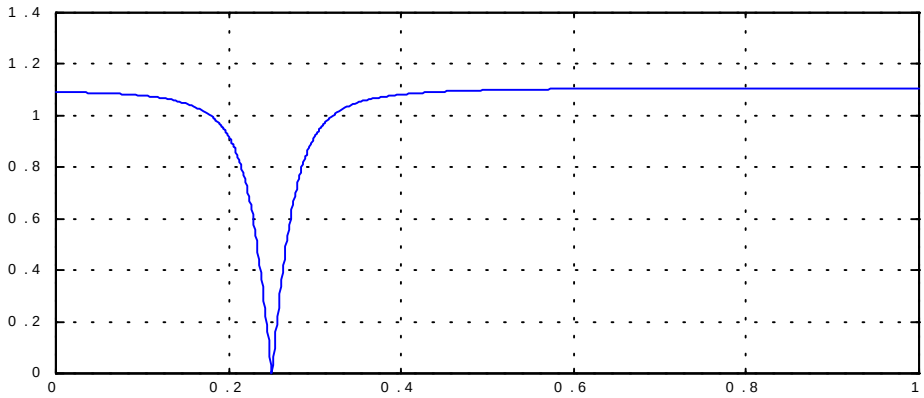
```
        out      = new_hist * coef(i,1) + history1 * coef(i,2) + history2 * coef(i,3) ;  % zeros
        hist(i,2) = hist(i,1);
        hist(i,1) = new_hist;
     end;
     y(j)=out;
 end;

subplot(2,1,1);plot([0:length(x)-1],x)

%axis([400,800,-2,2]);

title('iir: input');xlabel('n');ylabel('x(n)')

subplot(2,1,2);plot([0:length(y)-1],y)

%axis([0,1000,-2,2]);

title('iir: output ');xlabel('n');ylabel('y(n)')
pause;
sound(x);
pause;
sound(y);
```

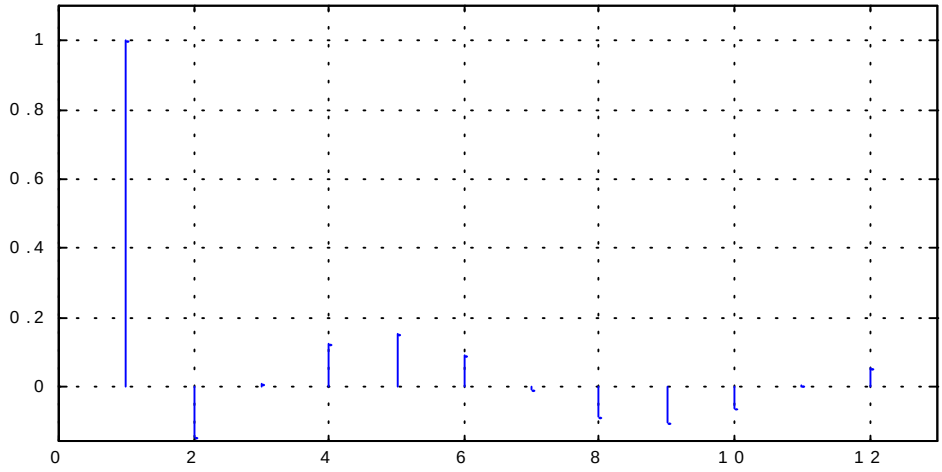
PRZYK£AD

```
fi=pi/4;
radius=0.9;
a=poly([radius*exp(j*fi), radius*exp(-j*fi)]) %zera i bieguny
b=poly([exp(j*fi), exp(-j*fi)])
```









PRZYKŁAD

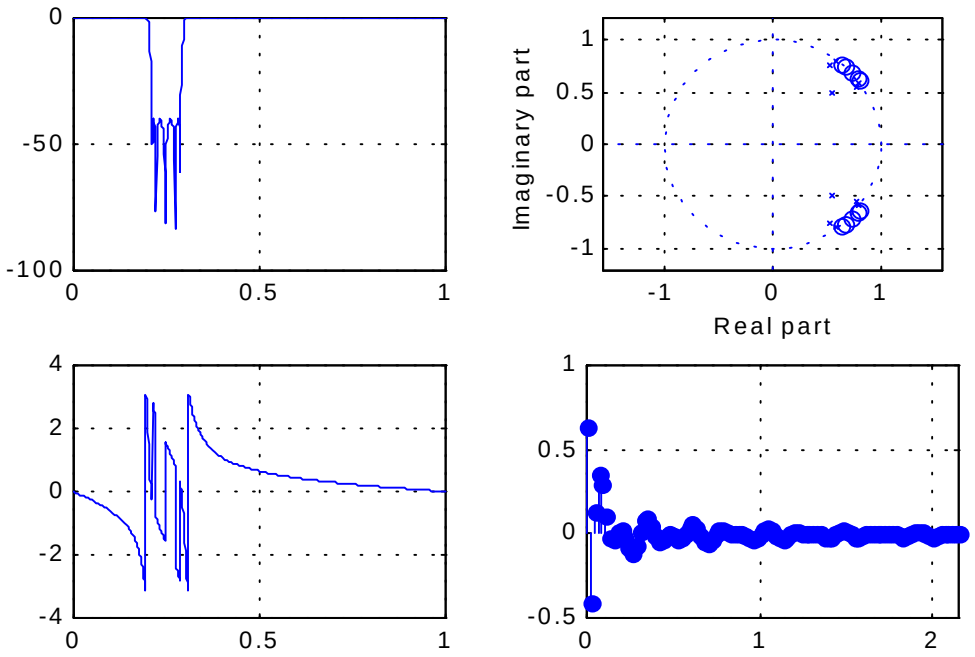
```
[b,a]    = ellip(5,0.3,40.2,[0.2,0.3],'stop' );
[z,p,k]  = ellip(5,0.3,40.2,[0.2,0.3],'stop' );

…
disp( b);
disp( a);
…
```

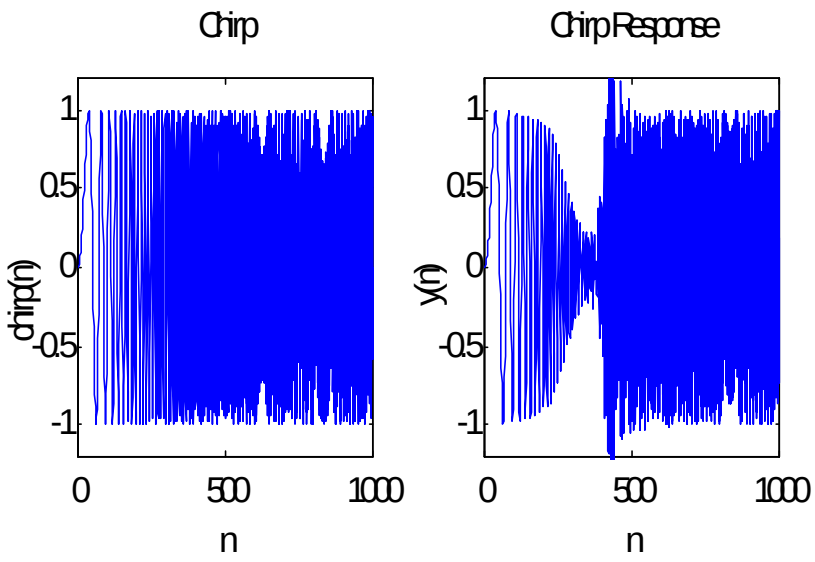
```
Columns 1 through 7

    0.6330   -4.4960   15.9138  -36.0239   57.3149  -66.6468   57.3149

  Columns 8 through 11

  -36.0239   15.9138   -4.4960    0.6330

  Columns 1 through 7

    1.0000   -6.4648   20.8114  -42.8734   62.1148  -65.8008   51.5541

  Columns 8 through 11

  -29.5076   11.8559   -3.0400    0.3871
```

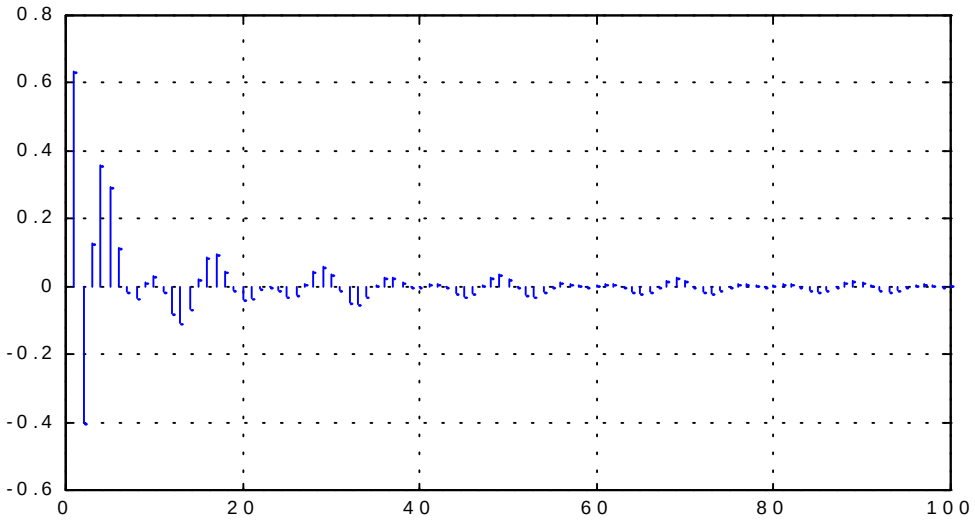
```
disp(sos);

    0.2959   -0.4660    0.2959    1.0000   -1.5977    0.9807
    1.1401   -1.4257    1.1401    1.0000   -1.1689    0.9737
    0.7967   -1.2226    0.7967    1.0000   -1.5452    0.8882
    1.4631   -1.9178    1.4631    1.0000   -1.0499    0.8438
    1.6100   -2.3052    1.6100    1.0000   -1.1032    0.5409

     5
```

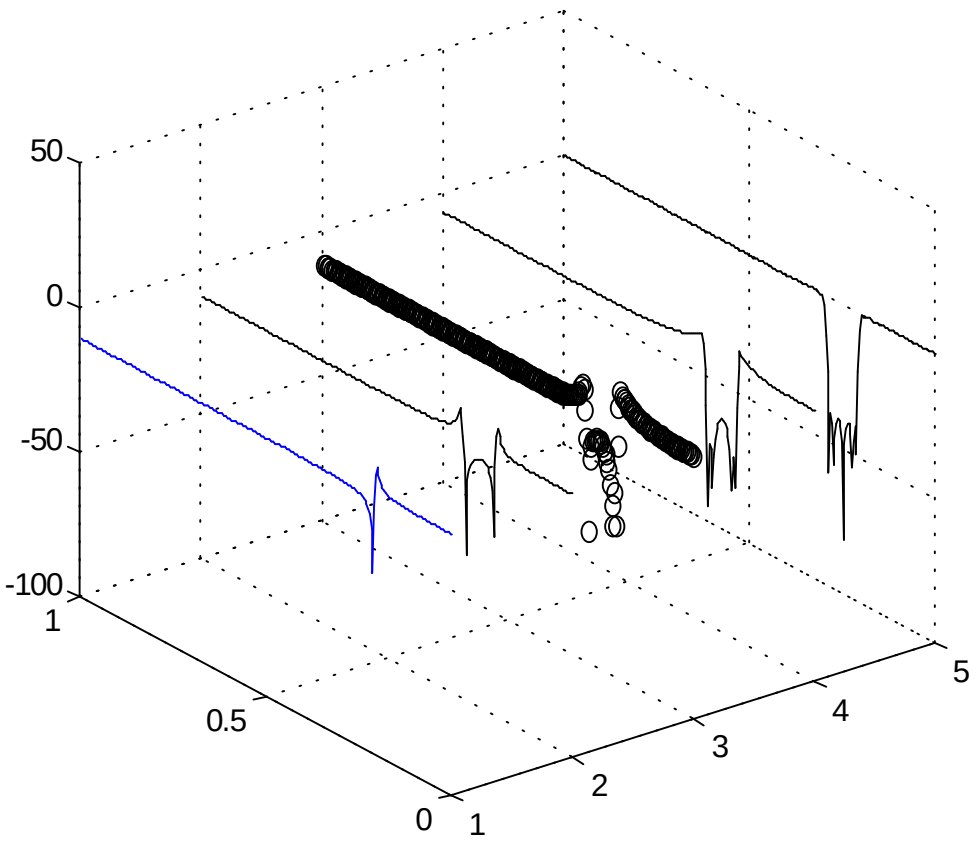
The output SOS is an L by 6 matrix which contains the coefficients of each second-order section in its rows:

```
SOS = [ b01 b11 b21  a01 a11 a21
        b02 b12 b22  a02 a12 a22
        ...
        b0L b1L b2L  a0L a1L a2L ]
```




Chirp
Chirp Response
chirp(n)
y(n)
1
0.5
0
-0.5
-1
0
500
1000
n


0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
20
40
60
80
100


50
0
-50
-100
1
0.5
0
1
2
3
4
5