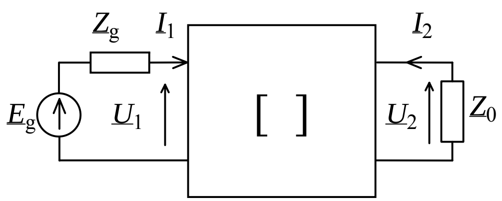
Tablica macierzy czwórnika

Ta macierz wyraża sie przez tę macierz następująco	**Y**	**Z**	**A**	**B**	**H**	**G**
Y	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\frac{1}{\det Z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$	$\frac{1}{a_{12}}\begin{bmatrix} a_{22} & -\det A \\ -1 & a_{11} \end{bmatrix}$	$\frac{1}{b_{12}}\begin{bmatrix} -b_{11} & 1 \\ \det B & -b_{22} \end{bmatrix}$	$\frac{1}{h_{11}}\begin{bmatrix} 1 & -h_{12} \\ h_{21} & \det H \end{bmatrix}$	$\frac{1}{g_{22}}\begin{bmatrix} \det G & g_{12} \\ -g_{21} & 1 \end{bmatrix}$
Z	$\frac{1}{\det Y}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\frac{1}{a_{21}}\begin{bmatrix} a_{11} & \det A \\ 1 & a_{22} \end{bmatrix}$	$-\frac{1}{b_{21}}\begin{bmatrix} b_{22} & 1 \\ \det B & b_{11} \end{bmatrix}$	$\frac{1}{h_{22}}\begin{bmatrix} \det H & h_{12} \\ -h_{21} & 1 \end{bmatrix}$	$\frac{1}{g_{11}}\begin{bmatrix} 1 & -g_{12} \\ g_{21} & \det G \end{bmatrix}$
A	$-\frac{1}{y_{21}}\begin{bmatrix} y_{22} & 1 \\ \det Y & y_{11} \end{bmatrix}$	$\frac{1}{z_{21}}\begin{bmatrix} z_{11} & \det Z \\ 1 & z_{22} \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$	$\frac{1}{\det B}\begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}$	$-\frac{1}{h_{21}}\begin{bmatrix} \det H & h_{11} \\ h_{22} & 1 \end{bmatrix}$	$\frac{1}{g_{21}}\begin{bmatrix} 1 & g_{22} \\ g_{11} & \det G \end{bmatrix}$
B	$\frac{1}{y_{12}}\begin{bmatrix} -y_{11} & 1 \\ \det Y & -y_{22} \end{bmatrix}$	$\frac{1}{z_{12}}\begin{bmatrix} z_{22} & -\det Z \\ -1 & z_{11} \end{bmatrix}$	$\frac{1}{\det A}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$	$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$	$\frac{1}{h_{12}}\begin{bmatrix} 1 & -h_{11} \\ -h_{22} & \det H \end{bmatrix}$	$\frac{1}{g_{12}}\begin{bmatrix} -\det G & g_{22} \\ g_{11} & -1 \end{bmatrix}$
H	$\frac{1}{y_{11}}\begin{bmatrix} 1 & -y_{12} \\ y_{21} & \det Y \end{bmatrix}$	$\frac{1}{z_{22}}\begin{bmatrix} \det Z & z_{12} \\ -z_{21} & 1 \end{bmatrix}$	$\frac{1}{a_{22}}\begin{bmatrix} a_{12} & \det A \\ -1 & a_{21} \end{bmatrix}$	$-\frac{1}{b_{11}}\begin{bmatrix} b_{12} & -1 \\ \det B & b_{21} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\frac{1}{\det G}\begin{bmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{bmatrix}$
G	$\frac{1}{y_{22}}\begin{bmatrix} \det Y & y_{12} \\ -y_{21} & 1 \end{bmatrix}$	$\frac{1}{z_{11}}\begin{bmatrix} 1 & -z_{12} \\ z_{21} & \det Z \end{bmatrix}$	$\frac{1}{a_{11}}\begin{bmatrix} a_{21} & -\det A \\ 1 & a_{12} \end{bmatrix}$	$-\frac{1}{b_{22}}\begin{bmatrix} b_{21} & 1 \\ -\det B & b_{12} \end{bmatrix}$	$\frac{1}{\det H}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$	$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$

Oznaczenia



Równania czwórnika

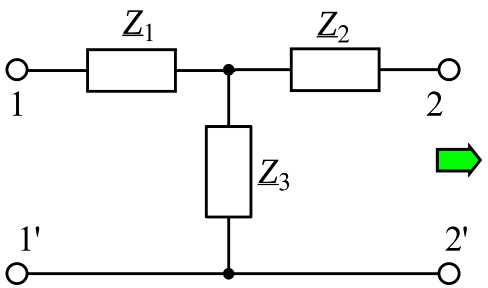
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} U_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} U_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} \qquad \begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}\begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

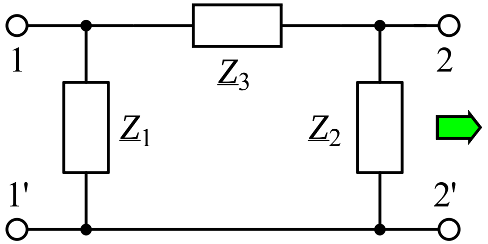
Parametry robocze czwórnika

impedancja wejściowa	$\underline{Z}_{wej} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{1+\underline{y}_{22}\underline{Z}_0}{\underline{y}_{11}+\det\underline{\mathbf{Y}}\,\underline{Z}_0} = \frac{\underline{a}_{11}\underline{Z}_0+\underline{a}_{12}}{\underline{a}_{21}\underline{Z}_0+\underline{a}_{22}}$,
impedancja wyjściowa	$\underline{Z}_{wyj} = \frac{\underline{U}_2}{\underline{I}_2} = \frac{1+\underline{y}_{11}\underline{Z}_g}{\underline{y}_{22}+\det\underline{\mathbf{Y}}\,\underline{Z}_g} = \frac{\underline{a}_{22}\underline{Z}_g+\underline{a}_{12}}{\underline{a}_{21}\underline{Z}_g+\underline{a}_{11}}$,
wzmocnienie napięciowe	$\underline{K}_u = \frac{\underline{U}_2}{\underline{U}_1} = \frac{-\underline{y}_{21}\underline{Z}_0}{1+\underline{y}_{22}\underline{Z}_0} = \frac{\underline{Z}_0}{\underline{a}_{11}\underline{Z}_0+\underline{a}_{12}}$,
wzmocnienie prądowe	$\underline{K}_i = \frac{\underline{I}_2}{\underline{I}_1} = \frac{\underline{y}_{21}}{\underline{y}_{11}+\det\underline{\mathbf{Y}}\,\underline{Z}_0} = \frac{-1}{\underline{a}_{21}\underline{Z}_0+\underline{a}_{22}}$,
skuteczne wzmocnienie napięciowe	$\underline{K}_{usk} = \frac{\underline{U}_2}{\underline{E}_g} = \frac{-\underline{y}_{21}\underline{Z}_0}{1+\underline{y}_{11}\underline{Z}_g+\underline{y}_{22}\underline{Z}_0+\det\underline{\mathbf{Y}}\,\underline{Z}_0\underline{Z}_g} = \frac{\underline{Z}_0}{\underline{a}_{12}+\underline{a}_{11}\underline{Z}_0+\underline{a}_{22}\underline{Z}_g+\underline{a}_{21}\underline{Z}_0\underline{Z}_g}$
skuteczne wzmocnienie mocy	$K_{psk} = \frac{P_2}{P_{g\,dys}} = 4\left\|\underline{K}_{usk}\right\|^2 \mathrm{Re}\left\{\frac{1}{\underline{Z}_0}\right\}\mathrm{Re}\left\{\underline{Z}_g\right\}$, $\quad P_2 = -\mathrm{Re}\left\{\underline{U}_2\underline{I}_2^*\right\}$, $\quad P_{g\,dys} = \frac{\left\|\underline{E}_g\right\|^2}{4\,\mathrm{Re}\left\{\underline{Z}_g\right\}}$

Ważne macierze czwórników pasywnych



$$\underline{Z} = \begin{bmatrix} \underline{Z}_1+\underline{Z}_3 & \underline{Z}_3 \\ \underline{Z}_3 & \underline{Z}_2+\underline{Z}_3 \end{bmatrix}$$



$$\underline{Y} = \begin{bmatrix} \frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_3} & -\frac{1}{\underline{Z}_3} \\ -\frac{1}{\underline{Z}_3} & \frac{1}{\underline{Z}_2}+\frac{1}{\underline{Z}_3} \end{bmatrix}$$

Warunki odwracalności i symetrii czwórnika

Założenie: istnieje macierz charakterystyczna	Czwórnik odwracalny	Czwórnik symetryczny
$\underline{\mathbf{Y}}$	$\underline{y}_{21} = \underline{y}_{12}$ $\underline{\mathbf{Y}}$ - macierz symetryczna	$\underline{y}_{21} = \underline{y}_{12}$ $\underline{y}_{22} = \underline{y}_{11}$
$\underline{\mathbf{Z}}$	$\underline{z}_{21} = \underline{z}_{12}$ $\underline{\mathbf{Z}}$ - macierz symetryczna	$\underline{z}_{21} = \underline{z}_{12}$ $\underline{z}_{22} = \underline{z}_{11}$
$\underline{\mathbf{A}}$	det $\underline{\mathbf{A}}$ = 1	det $\underline{\mathbf{A}}$ = 1 $\underline{a}_{22} = \underline{a}_{11}$
$\underline{\mathbf{B}}$	det $\underline{\mathbf{B}}$ = 1	det $\underline{\mathbf{B}}$ = 1 $\underline{b}_{22} = \underline{b}_{11}$
$\underline{\mathbf{H}}$	$\underline{h}_{21} = -\underline{h}_{12}$ $\underline{\mathbf{H}}$ - macierz skosnie symetryczna	$\underline{h}_{21} = -\underline{h}_{12}$ det $\underline{\mathbf{H}}$ =1
$\underline{\mathbf{G}}$	$\underline{g}_{21} = -\underline{g}_{12}$ $\underline{\mathbf{G}}$ - macierz skosnie symetryczna	$\underline{g}_{21} = -\underline{g}_{12}$ det $\underline{\mathbf{G}}$ =1

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