ETE0141 — odpowiedzi do zadań z zestawów

Zestaw 1.

Zad. 2.

a) $$F(s)=\frac{6(s+2)}{\left(s^2+4s+13\right)^2},$$

b) $$F(s)=\frac{5\left(s^4-4s^3+56s^2-100s+575\right)}{\left(s^2+25\right)^3},$$

c) $$F(s)=\frac{s^4+18s^2+648}{s^2\left(s^2+36\right)^2},$$

d) $$F(s)=\frac{\mathrm{e}^{-(s+2)}}{s+2},$$

e) $$F(s)=-\frac{s^3-3s^2+6s-6}{s^4}\mathrm{e}^{-s},$$

f) $$F(s)=\frac{1+\mathrm{e}^{-\pi s}}{\left(s^2+1\right)\left(1-\mathrm{e}^{-\pi s}\right)}=\frac{1}{\left(s^2+1\right)\operatorname{th}\frac{\pi s}{2}},$$

g) $$F(s)=\frac{1-\mathrm{e}^{-s}}{s^2\left(1+\mathrm{e}^{-s}\right)}=\frac{\operatorname{th}\frac{s}{2}}{s^2},$$

h) $$F(s)=\frac{\frac{2\pi}{T}\left(1+\mathrm{e}^{-s\frac{T}{2}}\right)}{s^2+\left(\frac{2\pi}{T}\right)^2}.$$

Zad. 3.

a) $$F(s)=\frac{2s+3-(10s+3)\mathrm{e}^{-2s}}{s^2},$$

b) $$F(s)=\frac{5-10(s+1)\mathrm{e}^{-2s}+5(2s+1)\mathrm{e}^{-4s}}{s^3}.$$

Zad. 4.

a) $$f(t)=\frac{1}{24}(2t-\sin 2t)\mathbf{1}(t),$$

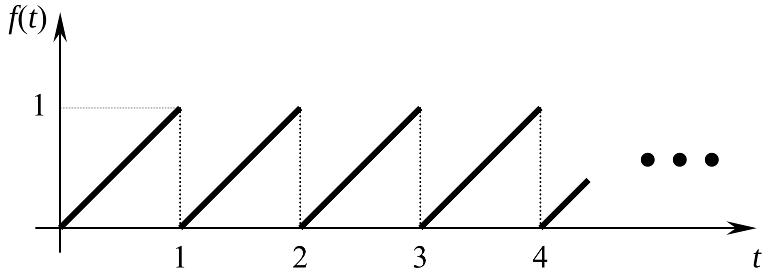
b) $$f(t)=\left(t-1+\mathrm{e}^{-t}\right)\mathbf{1}(t),$$

c) $$f(t)=\frac{1}{5}\left[\cos t+2\sin t-\mathrm{e}^{-t}\left(\cos t-3\sin t\right)\right]\mathbf{1}(t),$$

d) $$f(t)=\left(\cos t-\frac{1}{2}t\sin t\right)\mathbf{1}(t),$$

e) $$f(t)=\frac{1}{12}\left[8-(6t+9)\mathrm{e}^{-t}+\mathrm{e}^{-3t}\right]\mathbf{1}(t),$$

f)



g) $$f(t)=\mathrm{e}^{-t}(\cos t+\sin t)\mathbf{1}(t),$$

h) $$f(t)=\left[-2+\left(\frac{1}{2}t^2+3t+2\right)\mathrm{e}^{-t}\right]\mathbf{1}(t).$$

Zad. 5.

a) $$f(t)=\frac{\frac{E}{L}}{\sqrt{\frac{R^2}{L^2}-\frac{4}{LC}}}\left(\mathrm{e}^{-\frac{\frac{R}{L}-\sqrt{\frac{R^2}{L^2}-\frac{4}{LC}}}{2}t}-\mathrm{e}^{-\frac{\frac{R}{L}+\sqrt{\frac{R^2}{L^2}-\frac{4}{LC}}}{2}t}\right)\mathbf{1}(t),$$

b) $$f(t)=\frac{E}{L}t\mathrm{e}^{-\frac{t}{\sqrt{LC}}}\mathbf{1}(t),$$

c) $$f(t)=\frac{\frac{E}{L}}{\sqrt{\frac{1}{LC}-\frac{1}{2}\sqrt{\frac{R}{L}}}}\mathrm{e}^{-\frac{1}{2}\sqrt{\frac{R}{L}}t}\sin\sqrt{\frac{1}{LC}-\frac{1}{2}\sqrt{\frac{R}{L}}}\,t\cdot\mathbf{1}(t).$$

Zad. 6.

$$u(t)=2(t+1)\mathrm{e}^{-t}\mathbf{1}(t).$$

Zestaw 2.

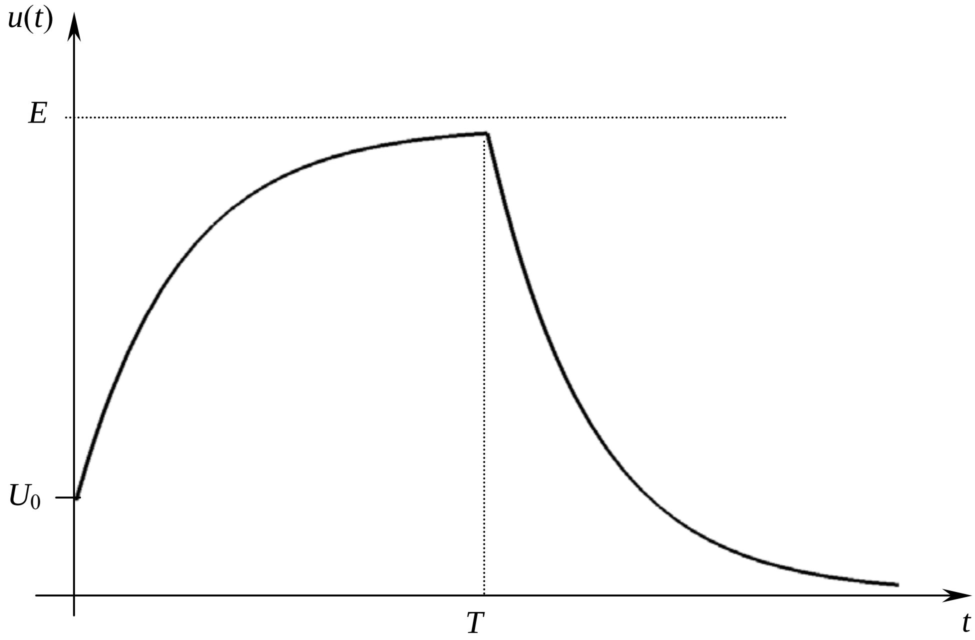
Zad. 1.

$$u(t)=12\cdot\mathbf{1}(t)+4\delta(t).$$

Zad. 2.

$$u(t)=E\left\{\left[1-\left(1-\frac{U_0}{E}\right)\mathrm{e}^{-\frac{t}{RC}}\right]\mathbf{1}(t)-\left(1-\mathrm{e}^{-\frac{t-T}{RC}}\right)\mathbf{1}(t-T)\right\}.$$

Przykładowy przebieg napięcia $u(t)$ przedstawiono na rysunku.



Zad. 3.

$$i_{C_2}(t)=\frac{1}{2}\left(3\mathrm{e}^{-t}-\mathrm{e}^{-\frac{1}{3}t}\right)\mathbf{1}(t)+3\left[1-\mathrm{e}^{-\frac{1}{3}(t-1)}\right]\mathbf{1}(t-1).$$

Zad. 4.

$$u(t)=3\left(\mathrm{e}^{-\frac{1}{5}t}-\mathrm{e}^{-t}\right)\mathbf{1}(t).$$

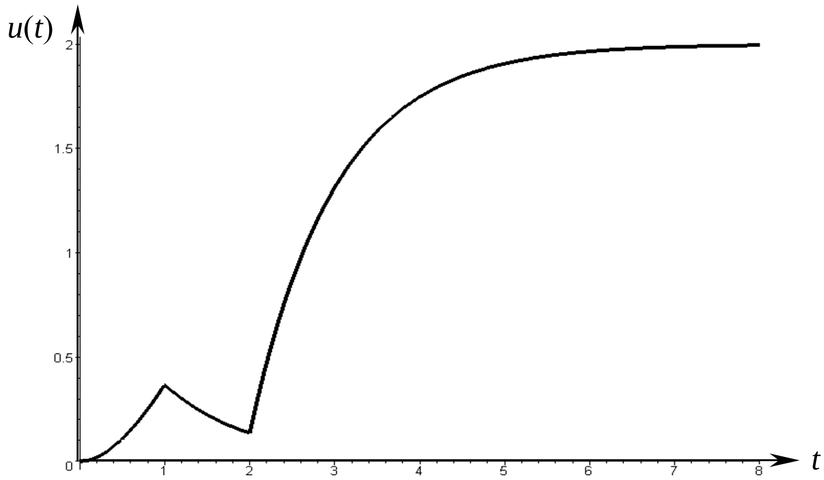
Zad. 5.

$$u(t)=\mathrm{e}^{-\frac{t}{2}}\sin\frac{t}{2}\cdot\mathbf{1}(t).$$

Zestaw 3.

Zad. 1.

$$u(t)=\left(t-1+\mathrm{e}^{-t}\right)\mathbf{1}(t)-(t-1)\mathbf{1}(t-1)+2\left[1-\mathrm{e}^{-(t-2)}\right]\mathbf{1}(t-2).$$



Zad. 2.

$$u(t)=6\left(2-t+\mathrm{e}^{-t}\right)\mathbf{1}(t)+6\left[t-2+\mathrm{e}^{-(t-1)}\right]\mathbf{1}(t-1).$$

Zad. 3.

$$i(t)=\underbrace{0,19255\mathrm{e}^{-158,73t}\mathbf{1}(t)}_{\text{Składowa przejściowa}}+1,2749\sin(314t+0,9916)\mathbf{1}(t)$$

Zad. 4.

$$i(t)=0,3\left(1-\mathrm{e}^{-250t}\right)\mathbf{1}(t).$$

Zad. 5.

$$\varphi=\operatorname{arctg}2\approx 1,1071.$$

Zestaw 4.

Zad. 1.

$$u(t)=\left(\mathrm{e}^{-4t}+4\cos 2t+2\sin 2t\right)\mathbf{1}(t).$$

Zad. 2.

$$u(t)=3\cos t\cdot\mathbf{1}(t).$$

Zad. 3.

a) $$h(t)=\left[51,9384\mathrm{e}^{-0,38197t}+0,061611\mathrm{e}^{-2,61803t}-(6t+52)\mathrm{e}^{-0,5t}\right]\mathbf{1}(t),$$

b) $$h(t)=\left(-0,03846\mathrm{e}^{-3,5t}-1,1\mathrm{e}^{-1,5t}+0,73846\cos 2t+1,50769\sin 2t\right)\mathbf{1}(t).$$

Zad. 4.

a) $$\mathrm{e}^{-2t}\mathbf{1}(t)*\mathrm{e}^{-3t}\mathbf{1}(t)=\left(\mathrm{e}^{-2t}-\mathrm{e}^{-3t}\right)\mathbf{1}(t),$$

b) $$\mathrm{e}^{-2t}\mathbf{1}(t)*\mathbf{1}(t)=\frac{1}{2}\left(\mathrm{e}^{-2t}-\mathrm{e}^{-3t}\right)\mathbf{1}(t),$$

c) $$\mathbf{1}(t)*\mathbf{1}(t)=t\mathbf{1}(t),$$

d) $$\left[\mathbf{1}\left(t+\frac{1}{2}\right)-\mathbf{1}\left(t-\frac{1}{2}\right)\right]*\left[\mathbf{1}\left(t+\frac{1}{2}\right)-\mathbf{1}\left(t-\frac{1}{2}\right)\right]=(t+1)\mathbf{1}(t+1)-2t\mathbf{1}(t)+(t-1)\mathbf{1}(t-1).$$

Zad. 5.

$$i(t)=\begin{cases}\dfrac{E}{t_0}\left(t-1+\mathrm{e}^{-t}\right) & \text{gdy } 0<t\le t_0\\[2mm] \dfrac{E}{t_0}\left[\mathrm{e}^{-t}+(t_0-1)\mathrm{e}^{-(t-t_0)}\right] & \text{gdy } t>t_0\end{cases}$$

co można przekształcić (jak?) do postaci:

$$i(t)=\frac{E}{t_0}\left\{\left(t-1+\mathrm{e}^{-t}\right)\mathbf{1}(t)-\left[t-1+(1-t_0)\mathrm{e}^{-(t-t_0)}\right]\mathbf{1}(t-t_0)\right\}.$$

Zad. 6.

Wielomianami Hurwitza są wielomiany a), f), h) oraz wielomiany g) gdy odpowiednio $ab > 0$ i $ac > 0$ (wielomian drugiego stopnia) i $ab > 0$, $ac > 0$, $ad > 0$, $bc - ad > 0$ (wielomian trzeciego stopnia).

Zad. 7.

Układ będzie stabilny w sensie BIBO gdy $k < 2{,}1$.

Zad. 8.

Układ będzie stabilny w sensie BIBO gdy $g > 2$.

Zad. 9.

Układ będzie stabilny w sensie BIBO gdy $k < 1$.

Zad. 10.

Układ będzie stabilny w sensie BIBO gdy $k < 1$.

Zad. 11.

$$T(s) = \frac{U_2(s)}{U_1(s)} = -\frac{s(s^2 - a)}{3s^3 + 4s^2 + (3-a)s + 1 - a}.$$

Układ będzie stabilny w sensie BIBO gdy $a < 1$.

Zad. 12.

$$\frac{U_2(s)}{U_1(s)} = \frac{\beta}{s^2C^2R^2 + sCR(3-\beta) + 1}.$$

Układ będzie stabilny w sensie BIBO gdy $\beta < 3$.