Prof. Zoran Vukic, PhD
Faculty of Electrical Engineering and Computing
University of Zagreb

Ognjen Kuljaca
Automation and Robotics Research Institute
The University of Texas at Arlington

LECTURES ON PID CONTROLLERS

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CONTENTS

1. CONVENTIONAL CONTROLLERS

Today, a number of different controllers are used in industry[1] and in many other fields. In quite general way those controllers can be divided into two main groups:

- conventional controllers
- unconventional controllers

As conventional controllers we can count a controllers known for years now, such as P, PI, PD, PID, Otto-Smith, all their different types and realizations, and other controller types[2]. It is a characteristic of all conventional controllers that one has to know a mathematical model of the process in order to design a controller. Unconventional controllers utilize a new approaches to the controller design in which knowledge of a mathematical model of a process generally is not required. Examples of unconventional controller are a fuzzy controller and neuro or neuro-fuzzy controllers.

Manny industrial processes are nonlinear and thus complicate to describe mathematically. However, it is known that a good many nonlinear processes can satisfactory controlled using PID controllers providing that controller parameters are tuned well. Practical experience shows that this type of control has a lot of sense since it is simple and based on 3 basic behavior types: proportional (P), integrative (I) and derivative (D). Instead of using a small number of complex controllers, a larger number of simple PID controllers is used to control simpler processes in an industrial assembly in order to automates the certain more complex process. PID controller and its different types such as P, PI and PD controllers are today a basic building blocks in control of various processes. In spite their simplicity, they can be used to solve even a very complex control problems, especially when combined with different functional blocks, filters (compensators or correction blocks), selectors etc. A continuous development of new control algorithms insure that the time of PID controller has not past and that this basic algorithm will have its part to play in process control in foreseeable future. It can be expected that it will be a backbone of many complex control systems.

1.1 Basic controller types

PID controllers use a 3 basic behavior types or modes: P - proportional, I - integrative and D - derivative. While proportional and integrative modes are also used as single control modes, a derivative mode is rarely used on it's own in control systems. Combinations such as PI and PD control are very often in practical systems. It can be also shown that PID controller is a natural generalization of a simplest possible controller - On-off controller.

[1] Pharmaceutical, chemical industry, etc.
[2] Optimal, adaptive, robust, nonlinear etc.

1.1.1 On-off controller

On-off controller algorithm is defined as:

$$u(t) = \begin{Bmatrix} U_{max}; \forall\, e(t) > 0 \\ U_{min}; \forall\, e(t) < 0 \end{Bmatrix}, \tag{1-1}$$

where:

e(t) – control error (for unit feedback)
u(t) – control signal (controller output).

Static characteristic of On-off controller is given in Fig. 1-1.

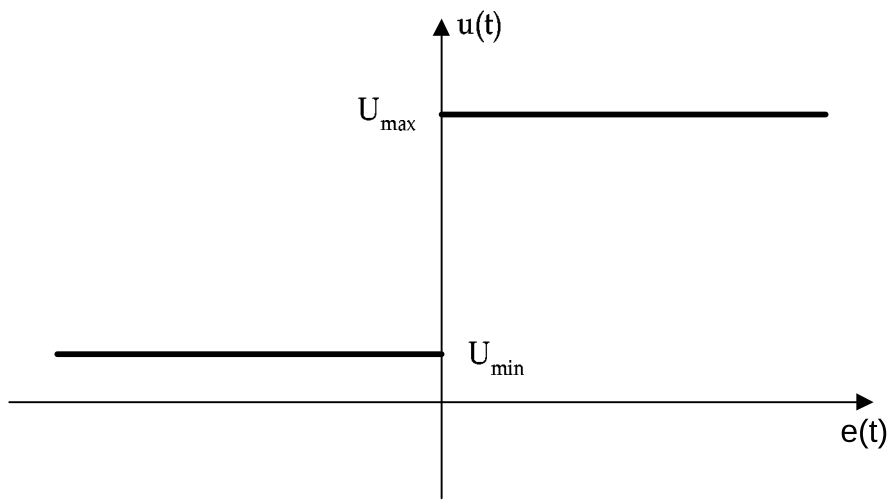


Fig. 1-1: Static characteristic of On-off controller

Control signal u(t) can have only two possible values, high U_{max} or low level U_{min}, depending if error is positive or negative.

Assuming that process (controlled plant) has a positive static gain, high-level control signal will cause increase in controlled variable value. The main idea in this way of control, with only two control levels is achieve desired value of the controlled variable in shortest time possible.

An inadequacy in this way of control is that control signal oscillates which may cause control variable to oscillate around desired value. Sometimes there is no remedy for this problem. For example, if level of liquid in tank is controlled using valve with only two possible states (open or closed) the level will always oscillates around desired value.

On-off controller is very simple since there are only two possible control signal values, no matter what is the value of control error. Process is forced to oscillate since u(t) is never zero (it is either U_{max} or U_{min}). The only way to avoid these forced oscillations is to diminish gain for small values of control error e(t). That can be achieved by introducing a proportional mode that will be active for certain values of control error[3].

[3] Nominal (operating) P mode area

1.1.2 P controller

P controller control algorithm is given with:

$$u(t)=\begin{Bmatrix} U_{max};\forall e(t)>e_0 \\ u_0+Ke(t)\ \text{for}\ -e_0<e(t)<e_0 \\ U_{min};\forall e(t)<-e_0 \end{Bmatrix}, \qquad (1\text{-}2)$$

where:

u_0 – amplitude of control signal when control error is equal 0
K – P controller gain for P mode nominal area $e(t)<|e_0|$

Many industrial controllers have defined a proportional band (PB) instead of gain:

$$PB=\frac{100}{K}[\%], \qquad (1\text{-}3)$$

It should be noted that for K=1 a proportional band is equal PB = 100%. Static characteristic of P controller is given in Fig. 1-2.

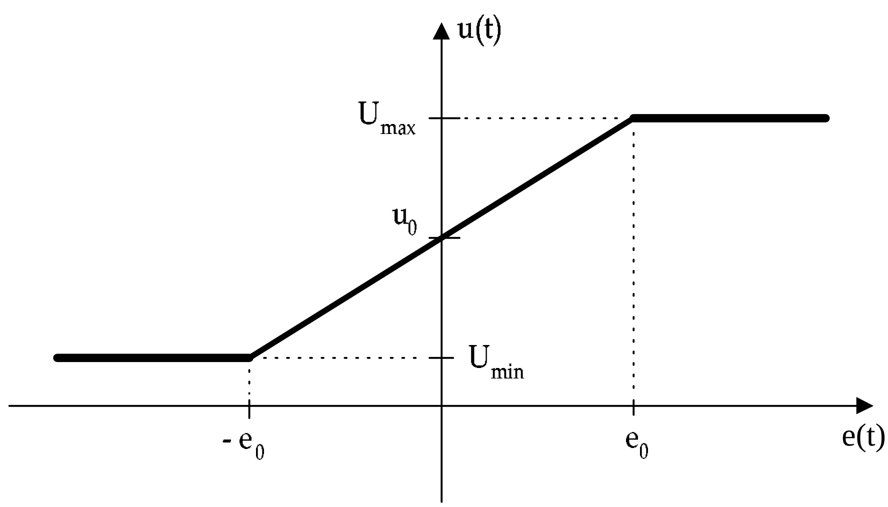


Fig. 1-2: Static characteristic of P controller

P controller can eliminate forced oscillations caused by use of on-off controller. However, a second problem arises. There exists now a steady state error. A relationship between control signal and error inside area $e(t)<|e_0|$ is given with:

$$u(t)=u_0+Ke(t). \qquad (1\text{-}4)$$

Error is then:

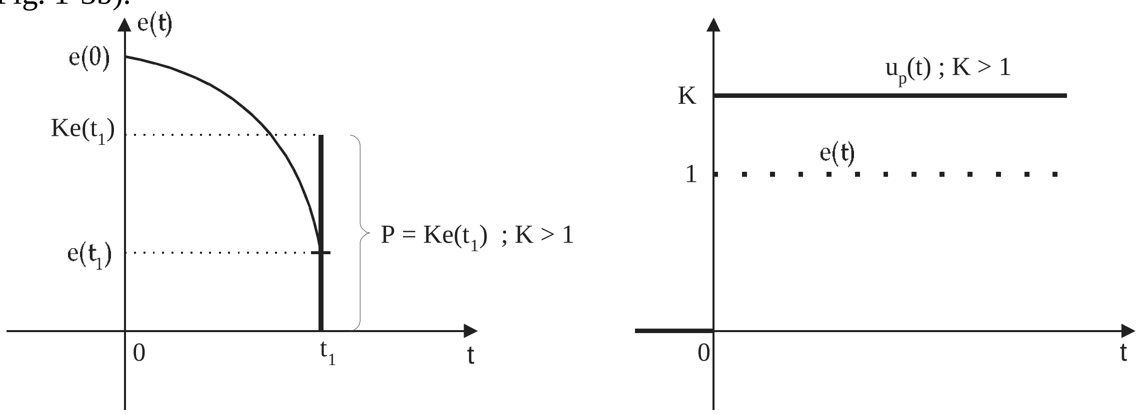
$$e(t) = \frac{u(t) - u_0}{K}. \qquad (1\text{-}5)$$

For a properly designed control system steady state error should be zero. With P controller that is possible if:

a) $K = \infty$
b) $u(t) = u_0$

The first alternative ($K = \infty$) cannot be physically realized in any proportional band (PB) excerpt for PB = 0 [%] which leads back to on-off controller and forced oscillations. The second alternative ($u(t) = u_0$) implies that it is possible to find u_0 at every moment and that it is possible to satisfy condition $u(t) = u_0$ for every given reference value r(t). This can be achieved if integral mode is added to P controller.

Proportional signal generation for P controller is shown in Fig. 1-3a) assuming $u_0 = 0$ and $K > 1$. P controller transfer function (unit step response) for $K > 1$ is shown in Fig. 1-3b).



a) Proportional signal generation

b) Transfer function

Fig. 1-3: Proportional signal generation and P controller transfer function

In general it can be said that P controller cannot stabilize higher order processes.

For the 1st order processes[4]., meaning the processes with one energy storage, a large increase in gain can be tolerated. Proportional controller can stabilize only 1st order unstable process. Changing controller gain K can change closed loop dynamics. A large controller gain will result in control system with:

a) smaller steady state error, i.e. better reference following
b) faster dynamics, i.e. broader signal frequency band of the closed loop system and larger sensitivity with respect to measuring noise
c) smaller amplitude and phase margin

[4] For strictly positive real (SPR) processes this claim stands for higher order than first also

1.1.3 PI controller

PI controller forms control signal in the following way:

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right], \qquad (1\text{-}6)$$

where:

T_i – integral time constant of PI controller

This is graphically shown in Fig. 1-4 assuming K = 1 and T_i = 1.

Constant $K_i = \dfrac{K}{T_i}$ is called "reset mode". Integral control is also sometimes called reset control.

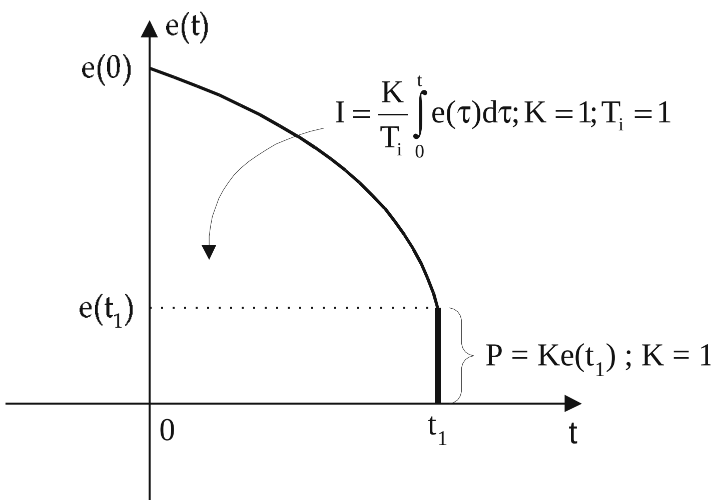


Fig. 1-4: PI controller signal generation

The name comes from the term "manual reset" which marks a manual change of operating point or of "bias" u_0 in order to eliminate error. PI controller performs this function automatically.

If control signal of P controller in proportional area is compared with PI controller output signal it can be seen that constant signal u_0 is replaced with signal proportional with the area under error curve:

$$u_0 = \frac{K}{T_i}\int_0^t e(\tau)d\tau\,. \qquad (1\text{-}7)$$

The fact that u_0 is replaced with an integral allows PI controller to eliminate steady state error. On the other hand, P controller cannot eliminate steady state error since it does not have any algorithm that would allow for the controller to increase control signal u(t) in order to increase controlled variable y(t) (assuming positive process gain) if in some moment t_1 error $e(t_1)$ = const. > 0. Proportional control law stays constant in this case and it will not try to change a controlled variable in such manner that control error is diminished.

PI controller on the other hand will increase control signal when error $e(t_1)$ = const. > 0. To the proportional part of the signal (P in Fig. 1-4) will be added integral part (I in Fig. 1-4) proportional to the area under curve e(t), so, overall signal

$$u(t) = Ke(t) + \frac{K}{T_i}\int_0^t e(\tau)d\tau = u_P(t) + u_I(t)\,, \qquad (1\text{-}8)$$

will be bigger.

Assuming positive process gain, increase in control signal will result in increase in controlled variable and error will tend toward zero.

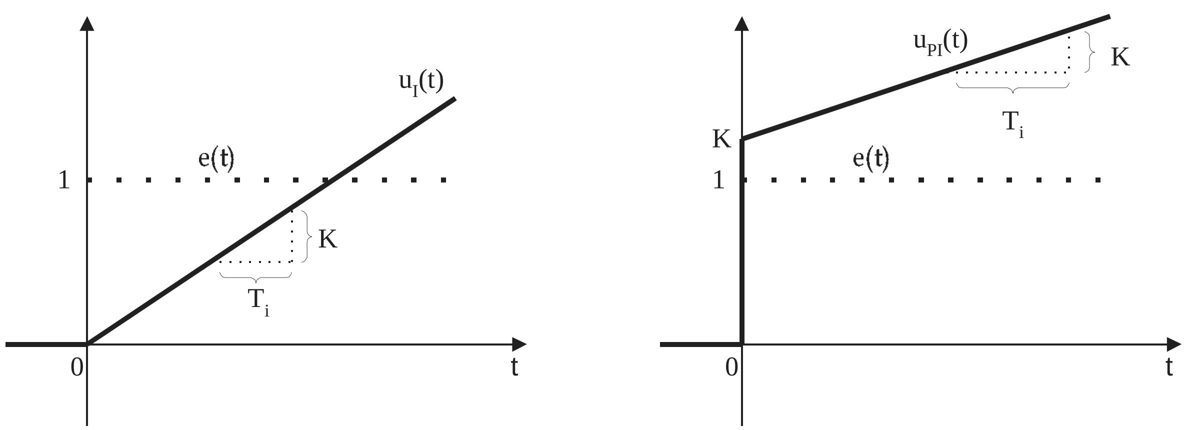
When e(t) < 0, control signal will decrease, control variable will also also decrease and error will tend toward zero. PI controller will not be active only when e(t) = 0. In all other situations PI controller will act to lead steady state control error to zero.

It can be concluded that PI controller will eliminate forced oscillations and steady state error resulting in operation of on-off controller and P controller respectively.

However, introducing integral mode has a negative effect on speed of the response and overall stability of the system.

PI controllers are very often used in industry, especially when speed of the response is not an issue.

Deceleration of response can be seen from transfer function of integrator shown in Fig. 1-5a).



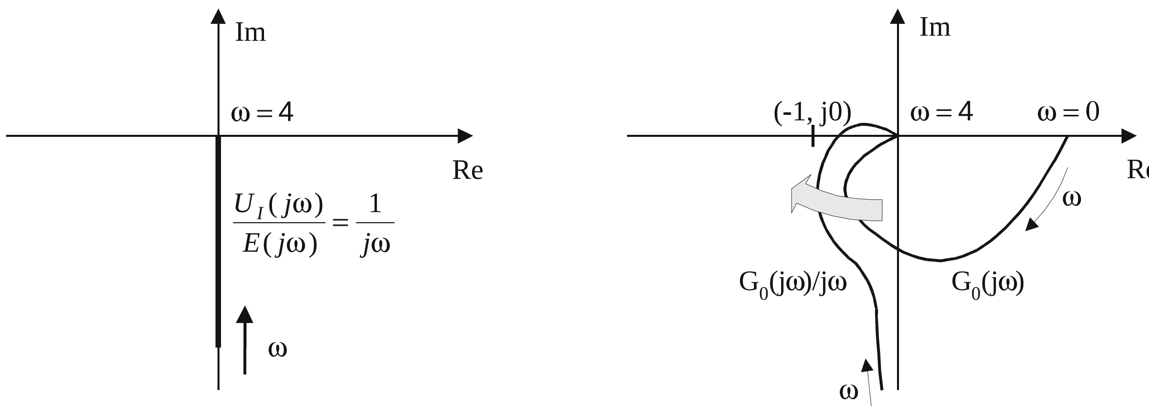
a) Transfer function of integrator

b) PI controller transfer function

Fig. 1-5: Transfer functions of integrator and PI controller

As it can be seen from Fig. 1-5 a sudden change in input signal (step) will result in gradual change of the output signal (ramp). Transfer function of PI controller is given in Fig. 1-5b). It can be seen that step change of the output is a result of proportional action, not integral.

Degradation of stability can be seen in frequency (Nyquist) characteristic where phase shift caused by integrator for all frequencies is - 90° (Fig. 1-6a)), thus the frequency characteristic moves closer to the critical point (-1, j0) (Fig. 1-6b)).

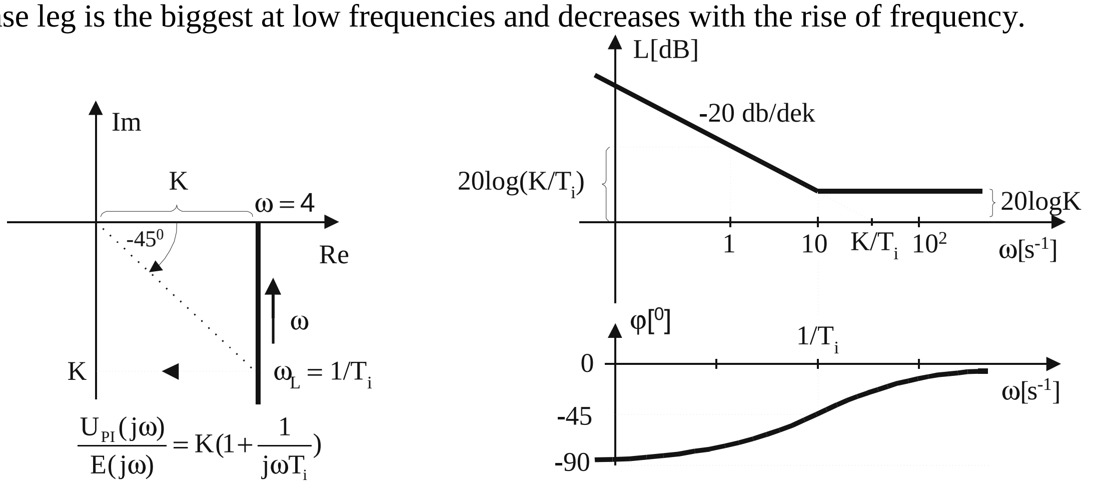


a) Frequency characteristic of integrator (Nyquist)

b) Destabilizing effect of integrator (Nyquist)

Fig. 1-6: Frequency characteristic and destabilizing effect of integrator

Frequency characteristic of PI controller is given in Fig. 1-7. It can be seen that phase lagging caused by PI controller is smaller than phase lag caused by pure integrator. Phase leg is the biggest at low frequencies and decreases with the rise of frequency.



a) Frequency characteristic of PI controller (Nyquist)

b) Frequency characteristic of PI controller (Bode)

Fig. 1-7: Frequency characteristic of PI controller

Thus, PI controller will not increase the speed of response. It can be expected since PI controller does not have means to predict what will happen with the error in near future. This problem can be solved by introducing derivative mode which has ability to predict what will happen with the error in near future and thus to decrease a reaction time of the controller.

Integral action can occur in the controller only on purpose, by design. Integral action can be noted on the other parts of the control system (actuators, plant etc.). These components may help in diminishing steady state error, but control system designer generally cannot tune this components.

1.1.4 PID controller

The role of derivative mode is illustrated in Fig. 1-8. It can be seen that two different situations are illustrated and one should expect different action from the controller. However, if PI controller is used the control signal will be the same in moment t_1 : $u(t_1)$. Proportional will be proportional to error in t_1:

$$u_P(t_1) = Ke(t_1)\,. \tag{1-9}$$

Integral part of the signal will be proportional to the area under error curve till moment t_1:

$$u_I(t_1) = \frac{K}{T_i}\int_0^{t_1} e(\tau)d\tau\,. \tag{1-10}$$

If $e(t_1)$ is the same in both cases, and if the area under error curve is the same, overall control signal in both cases will be the same. But, those two situations are different and required intervention should not be the same.

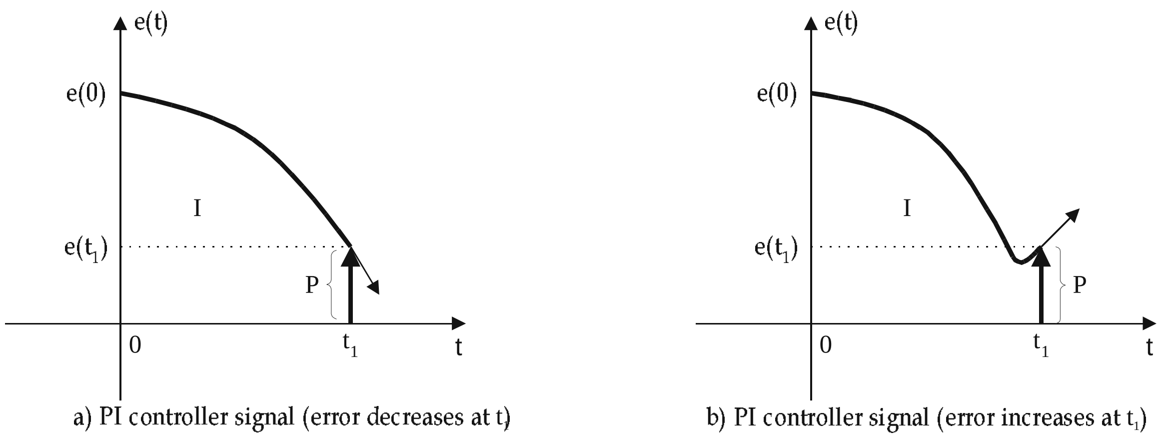


Fig. 1-8: PI controller output is the same for two different situations

In Fig. 1-8a) is illustrated a situation when error rapidly decreases. In that case a role of the controller is to decrease control signal in order to avoid possible control signal overshoot. In Fig. 1-8b) another situation is illustrated. After a sharp decrease the error start rising again. In this case controller has to react by increasing control signal in order to decrease the error.

This example shows a need for a controller that will generate control signal that will be also proportional to the error change (error trend). Derivative mode in PID controller fulfils that role.

Control signal of PID controller is:

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right], \tag{1-11}$$

or

$$u(t) = Ke(t) + K_i\int_0^t e(\tau)d\tau + K_d\frac{de(t)}{dt}, \tag{1-12}$$

where:

- $K_i = \dfrac{K}{T_i}$ - gain (reset) of integral part of the controller,
- $K_d = KT_d$ - gain of derivative part of the controller.

Derivative part of PID controller is proportional to the prognosis of error signal at time t + T_d where T_d is derivative time constant of the controller. In Fig. 1-9 PID controller control signal generation at time t_1 is illustrated.

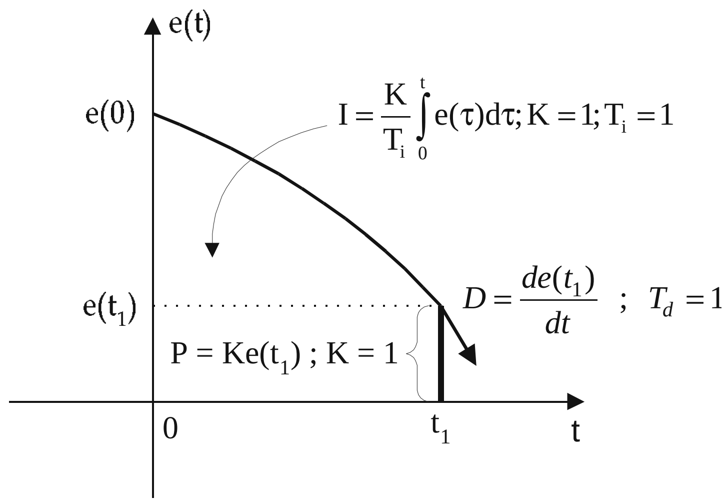
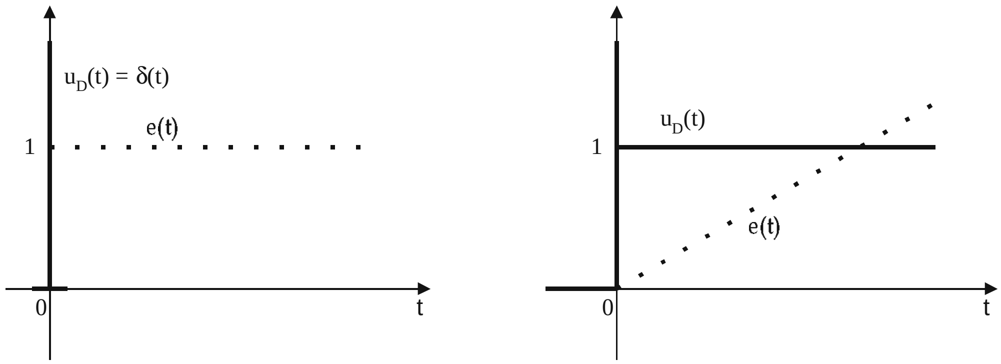


Fig. 1-9: PID controller control signal generation

Derivative mode is never used on it's own in the controller because derivative mode cannot eliminate control error. That fact can be seen in from transfer function of derivative element (Fig. 1-10). Derivative mode reacts only on change of the controller input. For ramp input derivative element will give a constant on its output as can be seen from Fig. 1-10b).



a) Transfer function of differentiator

b) D controller response on ramp input

Fig. 1-10: Transfer function and response on ramp input of derivative element

A transfer function of PID controller is obtained as sum of transfer functions of individual P, I and D elements (Fig. 1-11).

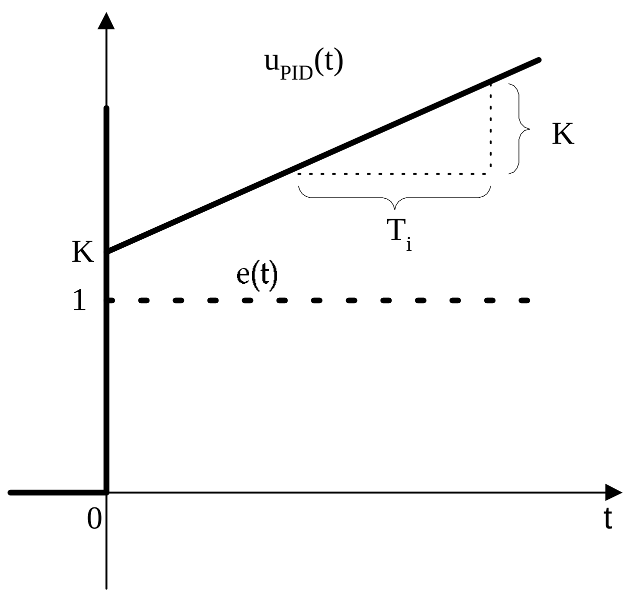
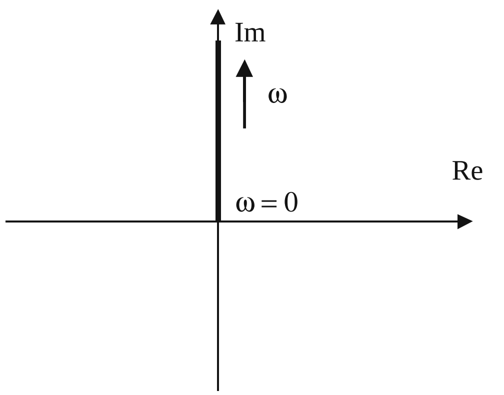


Fig. 1-11: Transfer function of PID controller

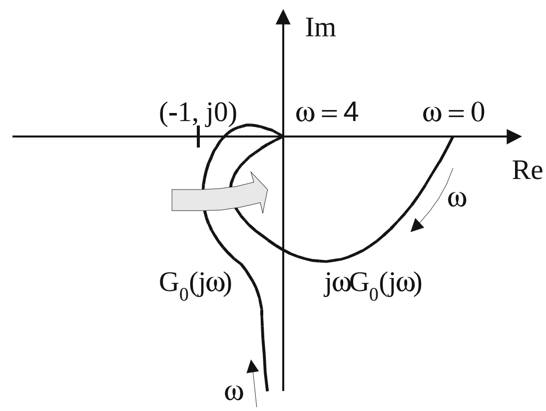
It can be concluded that PID controller has all the necessary dynamics: fast reaction on change of the controller input (D mode), increase in control signal to lead error towards zero (I mode) and suitable action inside control error area $e(t) < |e_0|$ to eliminate oscillations (P mode).

1.1.5 PD controller

D mode is used when prediction of the error can improve control or when it necessary to stabilize the system. From the frequency characteristic of D element it can be seen that it has phase lead of 90°. Thus, D element will move frequewncy characteristic of the open loop $G_o(j\omega)$ further away from the critical point (-1,j0) - Fig. 1-12b).



a) Frequency characteristic of differentiator (Nyquist)



b) Stabilizing effect of differentiator (Nyquist)

Fig. 1-12: Frequency characteristic and stabilizing effect of D element

Often derivative is not taken from the error signal but from the system output variable. This is done to avoid effects of the sudden change of the reference input that will cause sudden change in the value of error signal. Sudden change in error signal will cause sudden change in control output. To avoid that it is suitable to design D mode to be proportional to the change of the output variable y(t).

If there is a measuring noise present in y(t) will amplify this noise. Noise is usually higher frequency signal, so good remedy for the noise problem is use of low-pass filter in derivative channel that will insure derivative action only in the frequency band of interest and diminish negative effect of D mode on signal noise. Time constant of low-pass filter is often defined using derivative time constant of the controller as:

$$T_f = \frac{T_d}{N}, \qquad (1\text{-}13)$$

Majority of the controllers available at market today has N value between 3 and 20, which is satisfying in most situations.

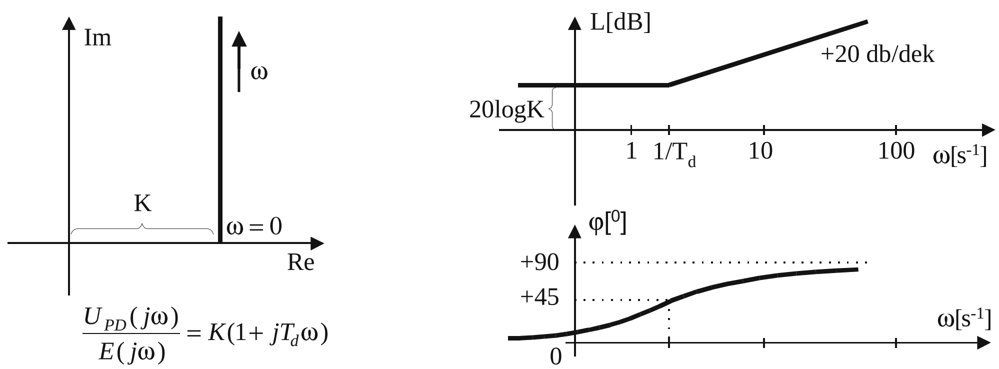
However, even with the use of low-pass filter one should be careful since remainder of the noise will be still amplified by derivative mode. derivative mode should be used only when noise is not significant or when controlled process reacts slowly on the change of error.

Filter is needed not only because of the effect of noise, but also because it is impossible to build ideal derivative elements since they are noncasual filters. Ideal D action is noncasual dynamics and it cannot be physically realized. Thus, instead of noncasual D mode in control is used casual derivative element (filter) such that:

$$sT_d \approx \frac{sT_d}{1+s\frac{T_d}{N}}, \qquad (1\text{-}14)$$

N is used to limit derivative gain on higher frequencies as can be seen from Fig. 1-14b).

Frequency characteristic of ideal (noncasual) PD controller is given in Fig. 1-13.



a) Frequency characteristic of ideal PD controller (Nyquist)

b) Frequency characteristic of ideal PD controller (Bode)

Fig. 1-13: Frequency characteristic of ideal PD controller

Transfer function of ideal PD controller is:

$$G_{PD}(s) = K(1+T_d s), \qquad (1\text{-}15)$$

Frequency characteristic is:

$$G_{PD}(j\omega) = K(1+j\omega T_d), \qquad (1\text{-}16)$$

Amplitude and phase characteristics are:

$$A(\omega) = K\sqrt{1+(\omega T_d)^2}, \qquad (1\text{-}17)$$

$$\varphi(\omega) = \mathrm{arctg}(\omega T_d), \qquad (1\text{-}18)$$

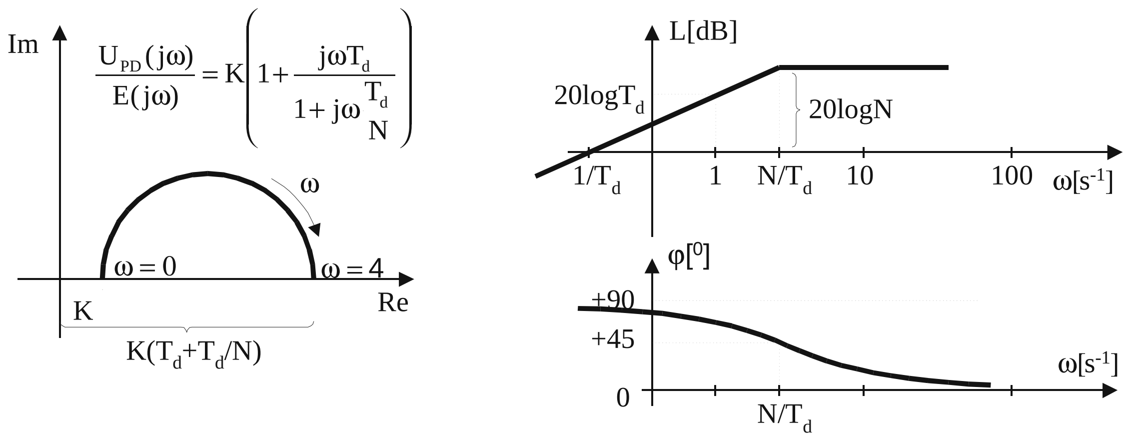
In time domain:

$$u_{sPD}(t) = K[1+T_d\delta(t)], \qquad (1\text{-}19)$$

Weighting function:

$$h_{PD}(t) = K[\delta(t) + T_d \frac{d\delta(t)}{dt}];\ t \geq 0\,, \tag{1-20}$$

Frequency characteristic of causal PD controller is given in Fig. 1-14.



a) Frequency characteristic of real PD controller (Nyquist)

b) Frequency characteristic of real PD controller (Bode)

Fig. 1-14: Frequency characteristic of casual PD controller

As it can be seen from Fig. 1-14b) N will limit gain at high frequencies. Stabilizing effect of PD controller can be seen form the phase lead. Phase lead is bigger on higher frequencies in the ideal case than in the causal case. In the causal case PD controller has lesser and lesser phase lead as frequency rises.

Processes that usually require control error prediction are thermal processes with big inertia. Speed of reaction in this case improves temperature control. Sometimes DPID controllers are used. In the case of DPID controllers control signal is proportional not only to the rate of change of process variable but also to the acceleration of change of process variable. However, these controllers can be used only if process has good filtering characteristics, (large inertia) since double derivation greatly amplifies noise.

When dealing with systems with transport delay it is also important to have a good error prediction. However, D mode will not be able to give a reliable prediction in the case of transport delay, so in those cases one should use Otto-Smith predictor (controller), not PID controller. If Otto-Smith predictor is not available it is better to use PI controller.

1.2 Choice of the controller type

Insofar were described proportional, integrative and derivative modes of the controllers and a rational behind their use was explained. However, excerpt for a few tips, an attention was not given to a question when to use different types of controllers. The rest of this section will give some answers on that particular topic.

1.2.1 On-off controller

On-off controller is the simplest controller and it has some important advontages. It is economical, simple to design and it does not require any parameter tuning. If oscillations will hamper the operation of the system and if controller parameter tuning is to be avoided, on-off controller is a good solution. In addition, if actuators work in only two modes (on and off), then it is almost always only controller that can be used with such actuators. That is a reason why on-off controllers are often used in home appliances (refrigerators, washers etc.) and in process industry when control quality requirements are not high (temperature control in buildings etc.). Additional advantage of on-off controllers is that they in general do not require any maintenance.

1.2.2 P controller

When P controller is used, large gain is needed to improve steady state error. Stable system do not have a problems when large gain is used. Such systems are systems with one energy storage (1^{st} order capacitive systems). If constant steady state error can be accepted with such processes, than P controller can be used. Small steady state errors can be accepted if sensor will give measured value with error or if importance of measured value is not too great anyway. Example of such system is liquid level control in tanks when exact approximate level of liquid suffice for the proper plant operation. Also, in cascade control sometime it is not important if there is an error inside inner loop, so P controller can a good solution in such cases.

Derivative mode is not required if the process itself is fast or if the control system as whole does not have to be fast in response. Processes of 1^{st} order react immediately on the reference signal change, so it is not necessary to predict error (introduce D mode) or compensate for the steady state error (introduce I mode) if it is possible to achieve satisfactory steady state error using only P controller.

1.2.3 PD controller

It is well known that thermal processes with good thermal insulation act almost as integrators. Since insulation is good and thermal losses are small, the most significant part of the energy that is led to the system is used temperature rise. Those processes allow for large gains so that integral mode in the controller is not needed. These processes can be described as different connections of thermal energy storages. Thermal energy is shifted from one storage into another. In general, with such processes there is present a process dynamics with large inertia. Since dynamics is slow, derivative mode is required for control of such processes. Integral mode would only already slow dynamics make more slowly. The other reason for using PPD controllers in such systems is that is possible to measure temperature with low level of noise in the measured signal.

PD controller is often used in control of moving objects such are flying and underwater vehicles, ships, rockets etc. One of the reason is in stabilizing effect of PD controller on sudden changes in heading variable y(t). Often a "rate gyro" for velocity measurement is used as sensor of heading change of moving object.

1.2.4 PI controller

PI controllers are the most often type used today in industry. A control without D mode is used when:

a) fast response of the system is not required
b) large disturbances and noise are present during operation of the process
c) there is only one energy storage in process (capacitive or inductive)
d) there are large transport delays in the system

If there are large transport delays present in the controlled process, error prediction is required. However, D mode cannot be used for prediction because every information is delayed till the moment when a change in controlled variable is recorded. In such cases it is better to predict the output signal using mathematical model of the process in broader sense (process + actuator). The controller structures that can be used are, for example, Otto-Smith predictor (controller) (Fig. 1-15), PIP controller or so called Internal Model Controller (IMC) (Fig. 1-16).

An interesting feature of IMC is that when the model of the process is precise ($A = A_M$ and $B = B_M$), then a feedback signal $e_M = y - y_M$ is equal to disturbance:

$$e_M = y - y_M = \frac{B}{A} u_{IMC} + d - \frac{B_M}{A_M} u_{IMC} = d, \qquad (1\text{-}21)$$

It follows that a control signal is not influenced by the reference signal and control systems behaves as open loop. A usual problems with stability that arrise when closed loop systems are used are then avoided.

Control system with IMC controller will be stable and if IMC and process are stable. With the exact model of process IMC is actually a feedforward controller and can designed as such, but, unlike feedforward controllers, it can compensate for unmeasured disturbances because feedback signal is equal to disturbance, which allows suitable tuning of the reference value of the controller.

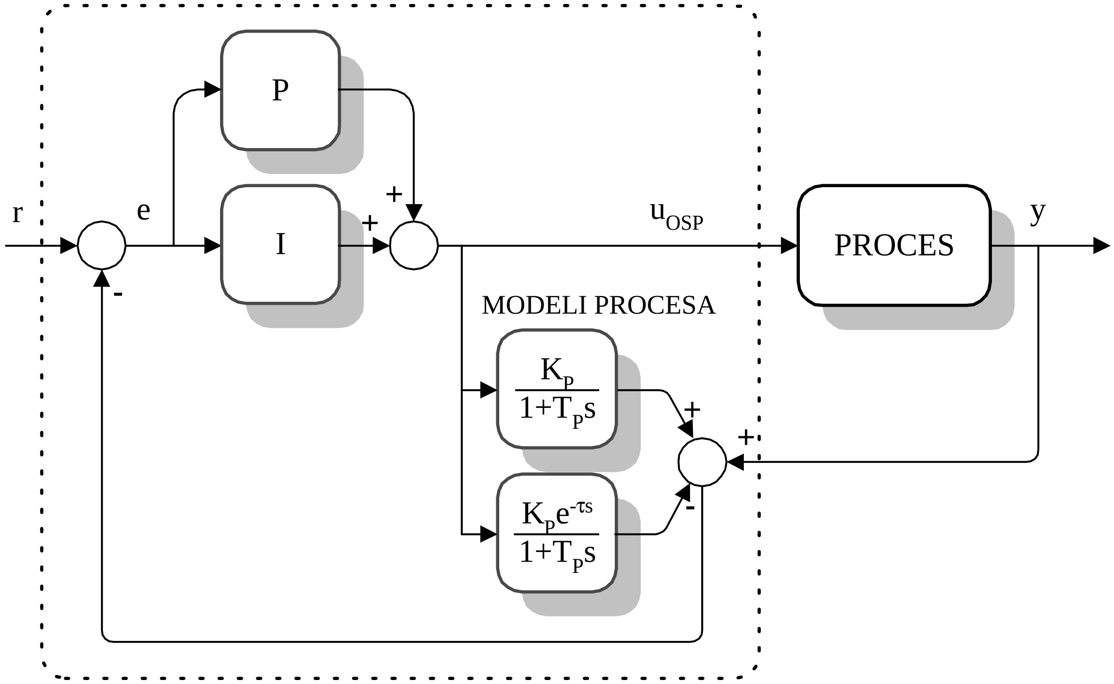


Fig. 1-15: Otto-Smith controller for systems with transport delay

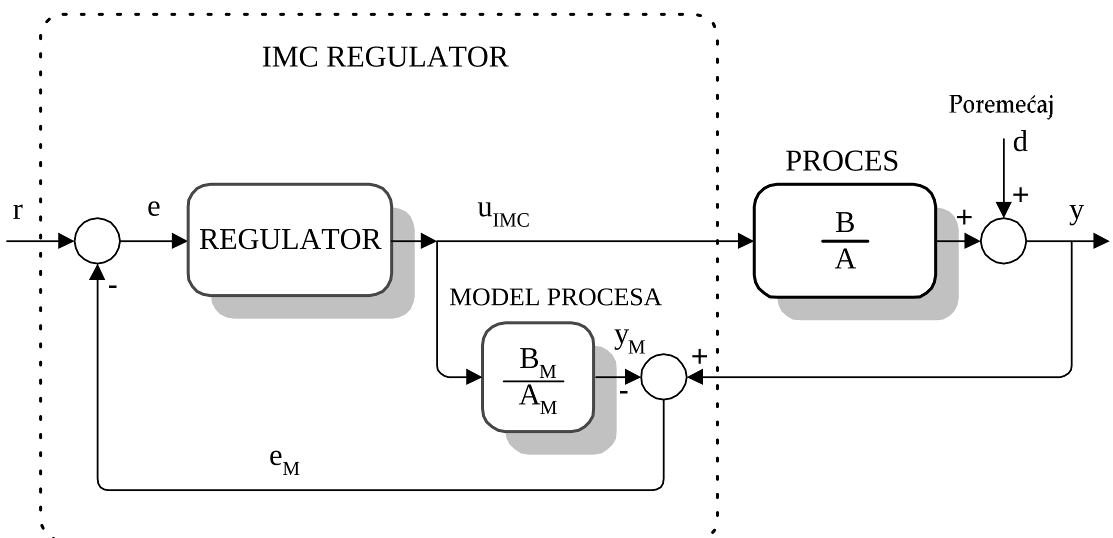


Fig. 1-16: Structure of IMC

If model of the process is not exact[5] ($A_M \neq A$, $B_M \neq B$), then feedback signal e_M will contain not only disturbance d but a modeling error $\left[\left(\frac{B}{A}-\frac{B_M}{A_M}\right)u_{IMC}\right]$ also. Thus, a

[5] That is usually so since at high frequencies usually it is not possible to describe the process dynamics precisely.

feedback will have its usual role, and stability problem can arise. This requires for parameters[6] to be tuned again so the stability is not lost.

1.2.5 PID controller

Derivative mode improves stability of the system and enables increase in gain K and decrease in integral time constant T_i, which increases speed of the controller response. PID controller is used when dealing with higher order capacitive processes (processes with more than one energy storage) when their dynamic is not simillar to the dynamics of an integrator (like in many thermal processes). PID controller is often used in industry, but also in the control of mobile objects (course and trajectory following included) when stability and precise reference following are required. conventional autopilot are for the most part PID type controllers.

1.3 Topology of PID controllers

Problem of topology (structure) of controller arises when:

- designing control system (defining structure and controller parameters)
- tuning parameters of the given controller

There are a number of different PID controller structures. Different manufacturers design controllers in different manner. However, two topologies are the most often case:

- parallel (non-interactive)
- serial (interactive)

Parallel structure is most often in textbooks, so it is often called "ideal" or "textbook type". This non-interactive structure because proportional, integral and derivative mode are independent on each other. Parallel structure is still very rare in the market. The reason for that is mostly historical. First controllers were pneumatic and it was very difficult to build parallel structure using pneumatic components. Due to certain conservatism in process industry most of the controller used there are still in serial structure, although it is relatively simple to realize parallel structure controller using electronics. In other areas, where tradition is not so strong, parallel structure can be found more often.

[6] Assuming the parameters were designed for operation in feedforward configuration

1.3.1 Parallel PID topology

A parallel connection of proportional, derivative and integral element is called parallel or non-interactive structure of PID controller. Parallel structure is shown in Fig. 1-17.

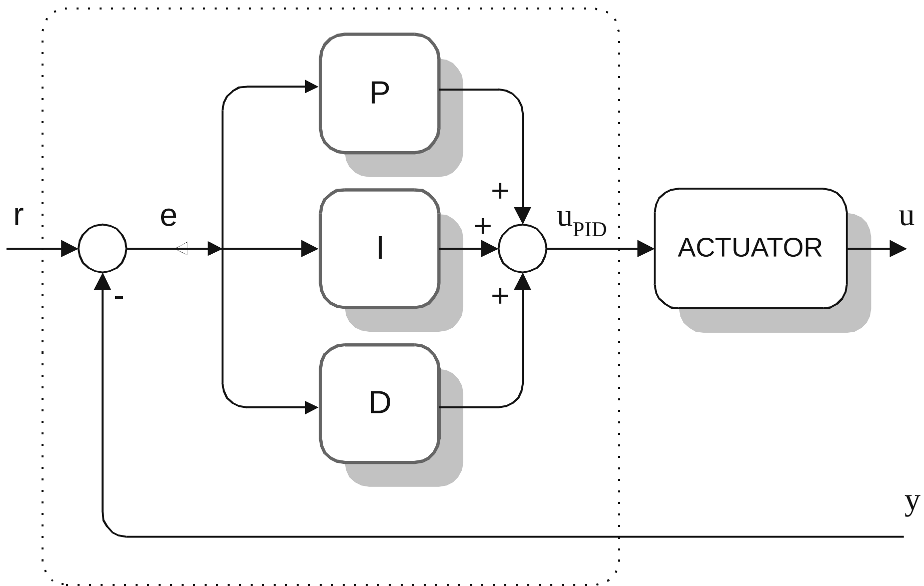


Fig. 1-17: Parallel structure of PID controller

PID controller algorithm is given by:

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right], \qquad (1\text{-}22)$$

or

$$u(t) = Ke(t) + K_i\int_0^t e(\tau)d\tau + K_d\frac{de(t)}{dt}. \qquad (1\text{-}23)$$

It can be seen that P, I and D channels react on the error signal and that they are unbundled. This is basic structure of PID controller most often found in textbooks. There are other non-interactive structures.

1.3.2 Non-interactive "Derivative-of-output controller form" (PI-D form)

Because of possible discontinuity (step change) in reference signal that are transferred into error signal and result in impulse traveling through derivative channel and thus cause large control signals u_{PID}, it is more suitable in practical implementation to use "derivative of output controller form". It is even more suitable controller structure if there exist sensors that give that information, such tachometers in electromechanical servo systems or "rate gyro" in mobile objects control. If PI-D structure (Fig. 1-18) is used, discontinuity in r(t) will be still transferred through proportional into control signal u_{PI-D}, but it will not have so strong effect as if it was amplified by derivative element.

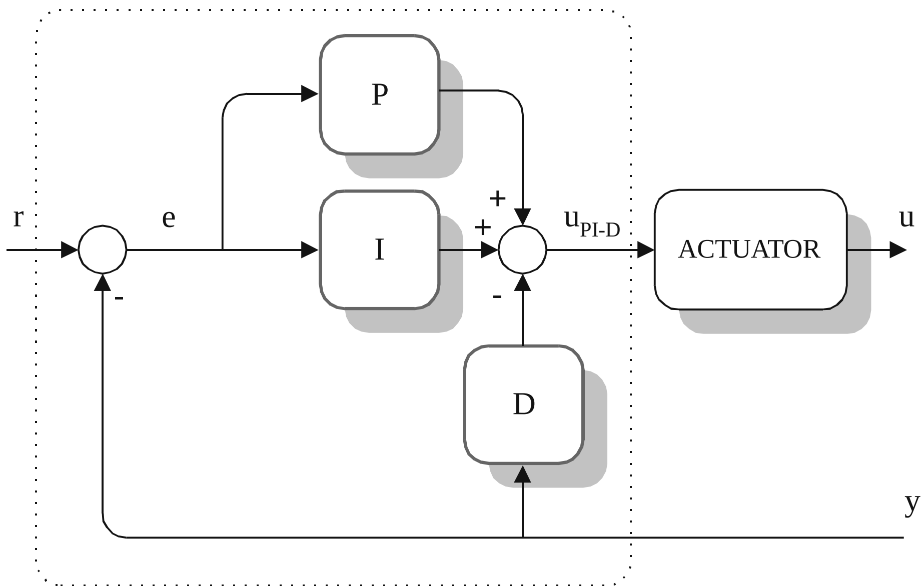


Fig. 1-18: Derivative of output controller form (PI-D form)

PI_D controller algorithm is given by:

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau - T_d\frac{dy(t)}{dt}\right], \qquad (1\text{-}24)$$

or

$$u(t) = Ke(t) + K_i\int_0^t e(\tau)d\tau - K_d\frac{dy(t)}{dt}. \qquad (1\text{-}25)$$

1.3.3 Standard form (ISA form)

Standard form takes care of possible discontinuity transfer through proportional and derivative channel. A weighting factor is used to limit transferred discontinuity. Also, instead ideal derivate a real derivate is used (casual). ISA form is shown in Fig. 1-19.

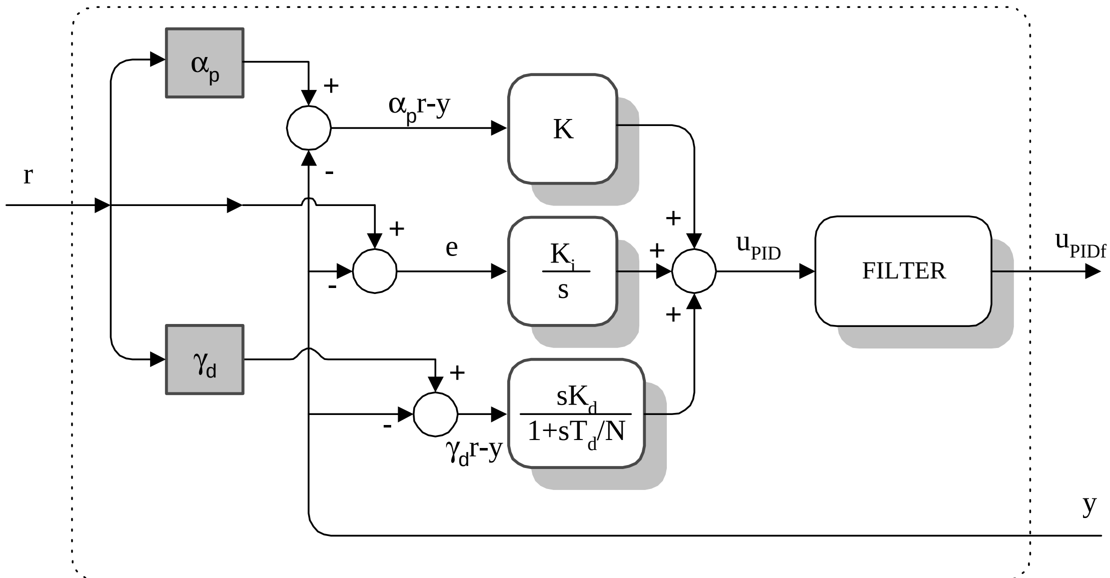


Fig. 1-19: Standard (ISA) form

ISA form algorithm is given as:

$$U_{ISA}(s) = K\left\{[\alpha_p R(s) - Y(s)] + \frac{1}{sT_i}E(s) + \frac{sT_d}{1+s\frac{T_d}{N}}[y_d R(s) - Y(s)]\right\}, \qquad (1\text{-}26)$$

or

$$U_{ISA}(s) = K[\alpha_p R(s) - Y(s)] + K_i\frac{1}{s}E(s) + \frac{sK_d}{1+s\frac{T_d}{N}}[y_d R(s) - Y(s)], \qquad (1\text{-}27)$$

Filter is usually used to filter out high frequency components form the controller output in order to spare actuator from unwarranted action. If sensor gives signals that cannot be followed by system, often a dead zone or notch filter is used instead of low-pass filter to spare actuator of the actions that will be of no use anyway.

Filter can be use with each of PID structures shown if it will improve control system performance. Type of the filter depends on actual case.

1.3.4 Set-point-on-I-only controller (I-PD form)

This structure of PID controller is not so often as PI-D structure, but it has certain advantages. Control law for this structure is given as:

$$u(t) = K\left[-y(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau - T_d\frac{dy(t)}{dt}\right], \tag{1-28}$$

or

$$u(t) = -Ky(t) + K_i\int_0^t e(\tau)d\tau - K_d\frac{dy(t)}{dt}. \tag{1-29}$$

Block diagram for I-PD form is shown in Fig. 1-20:

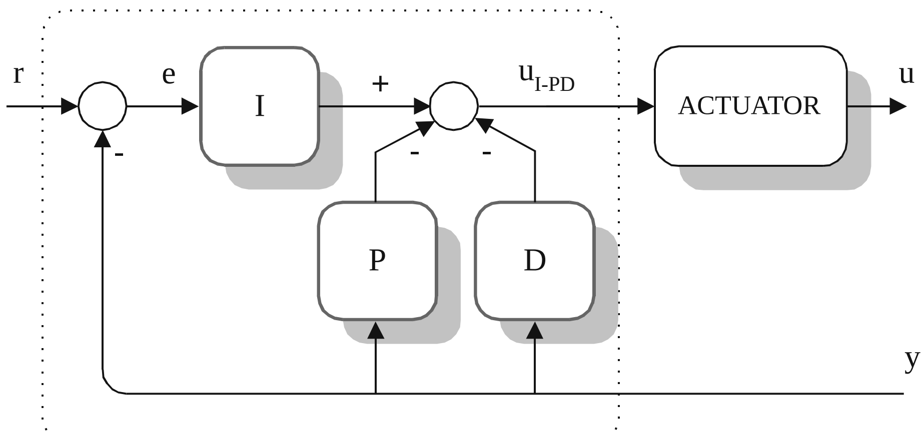


Fig. 1-20: I-PD forma of PID controller

With this structure transfer of reference value discontinuities to control signal is completely avoided. Control signal has less sharp changes than with other structures.

1.3.5 General structure of parallel PID controller

After the previous analysis a structure that can perform as any of the previously described controllers can be synthesized. A so called general structure of parallel PID controller is shown in . By defining different weighting factors different controller action could be realized.

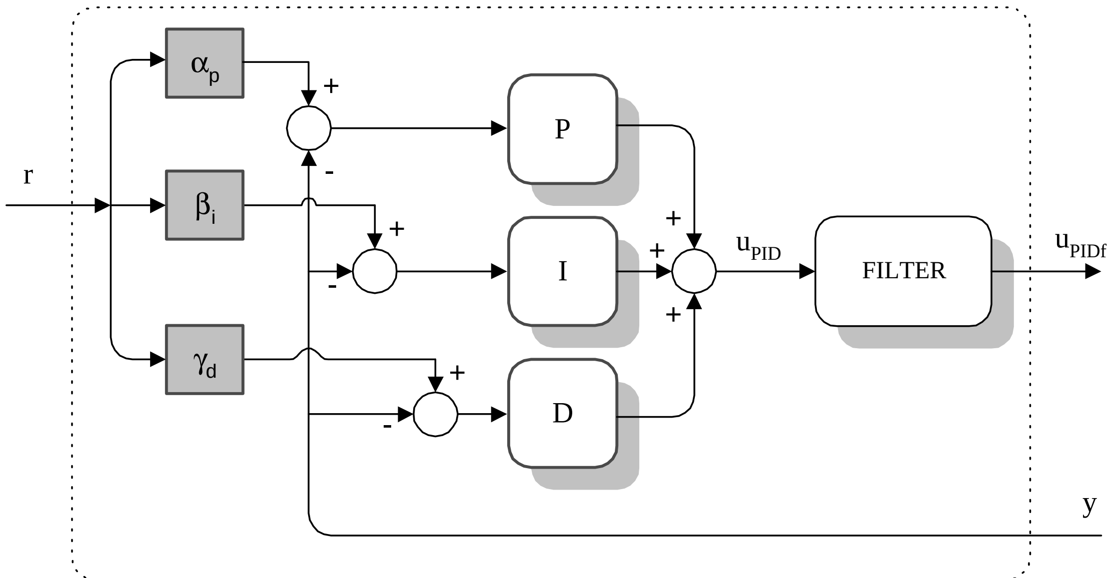


Fig. 1-21: General structure of PID controller

Weighting factors most often have the following values:

- α_p = 0 or 1
- β_i = 1
- γ_d = 0

1.3.6 Two-parameter form of PID controller

Parallel PID controller can be also represented in two-parameter form as in Fig. 1-22.

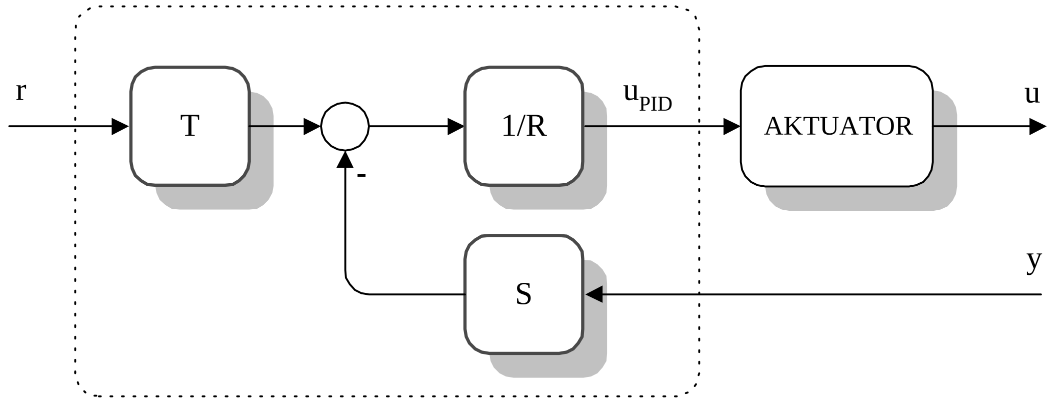


Fig. 1-22: Parallel PID controller in two-parameter form

where:

$T(s) = 1$
$R(s) = s(1+sT_f)$
$S(s) = k(s-z_1)(s-z_2)$; $k = K_d + KT_f$

In this two-parameter a casual D element is used. With ideal D element a transfer function would be:

$$G_{PID}(s) = \frac{K_d s^2 + Ks + K_i}{s} = \frac{K_d(s^2 + as + b)}{s} = \frac{K_d(s - z_1)(s - z_2)}{s}. \qquad (1\text{-}30)$$

where:

$$a = \frac{K}{K_d}$$
$$b = \frac{K_i}{K_d}$$
$z_1 + z_2 = -a$
$z_1 z_2 = b$

Polynomials R, S and T are now:

$T(s) = 1$
$R(s) = s$
$S(s) = K_d(s-z_1)(s-z_2)$

1.3.7 Serial (interactive) structure (PD*PI form)

This structure is very often in process industry. I channel uses both the error signal e(t) and derivative of the error signal $\frac{de(t)}{d(t)}$. It is realized as serial connection of PD and PI controller. Control algorithm is given as:

$$u_{PD*PI}(t) = K^s\left[e_1(t) + \frac{1}{T_i^s}\int_0^t e_1(\tau)d\tau\right], \tag{1-31}$$

where:

$$e_1(t) = e(t) + T_d^s\frac{de(t)}{dt}. \tag{1-32}$$

Block diagram is given in Fig. 1-23.

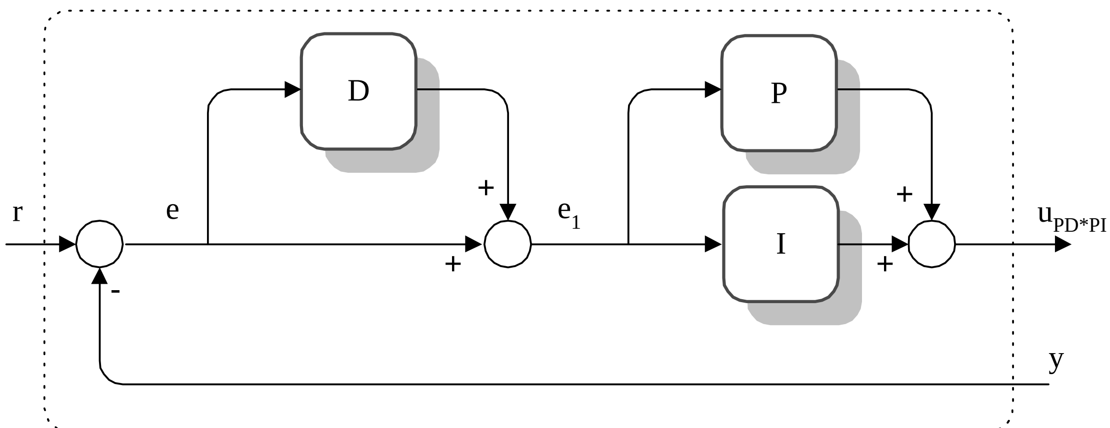


Fig. 1-23: Serial (interactive) control structure

1.3.8 Interconnection between parallel and serial structure

If parameters of serial structure are known, then it is possible to compute parameters of corresponding parallel structure using the following expressions:

$$K = K^s\frac{T_i^s + T_d^s}{T_i^s}; T_i = T_i^s + T_d^s; T_d = \frac{T_i^s T_d^s}{T_i^s + T_d^s}. \tag{1-33}$$

However, if parameters of the parallel structure are known, it is not always possible to compute corresponding serial structure. It will be possible to do that only if:

$$T_i > 4T_d \,. \tag{1-34}$$

The fact that this condition exists shows that the parallel structure is more general than serial structure. In most cases condition (1-34) is satisfied and in this case serial structure parameters can be computed from:

$$K^s = \frac{K}{2}\left(1+\sqrt{1-\frac{4T_d}{T_i}}\right);\ T_i^s = \frac{T_i}{2}\left(1+\sqrt{1-\frac{4T_d}{T_i}}\right);\ T_d^s = \frac{T_i}{2}\left(1-\sqrt{1-\frac{4T_d}{T_i}}\right). \tag{1-35}$$

Serial and parallel structures are different only in PID controller case. For P, PI and PD controllers both structures are identical.

1.4 PID controller parameter and topology identification

Problem of identification of PID controller arises when parameters of an existing controller have to be tuned. Manufacturers usually don't give data about controller structure (serial or parallel), so its structure also has to be determined. Controller parameters have to be manually tuned if they are changed with time (that is often with hydraulic and pneumatic controllers) or because process parameters have changed so the controller does not perform satisfactory anymore.

Not knowing the exact structure of the controller is not critical if manual parameter tuning can be done in controlled environment using trial and error method and if rules given in Table 1 are observed.

Parameter	Speed of response	Stability	Accuracy
increasing K	increases	deteriorate	improves
increasing K_i	decreases	deteriorate	improves
increasing K_d	increases	improves	no impact

Table 1 Rules for tuning PID controller parameters

However, if parameters are not tuned manually or if it is not possible to use trial and error method, then it is critical to know the controller structure. Identification can be performed experimentally using a certain type of reference signal on the controller input (signal y(t)) and measuring response on the controller output ($u_{PID}(t)$) - Fig. 1-24. From the controller response it is possible to draw conclusions about parameters and structure of the controller under test.

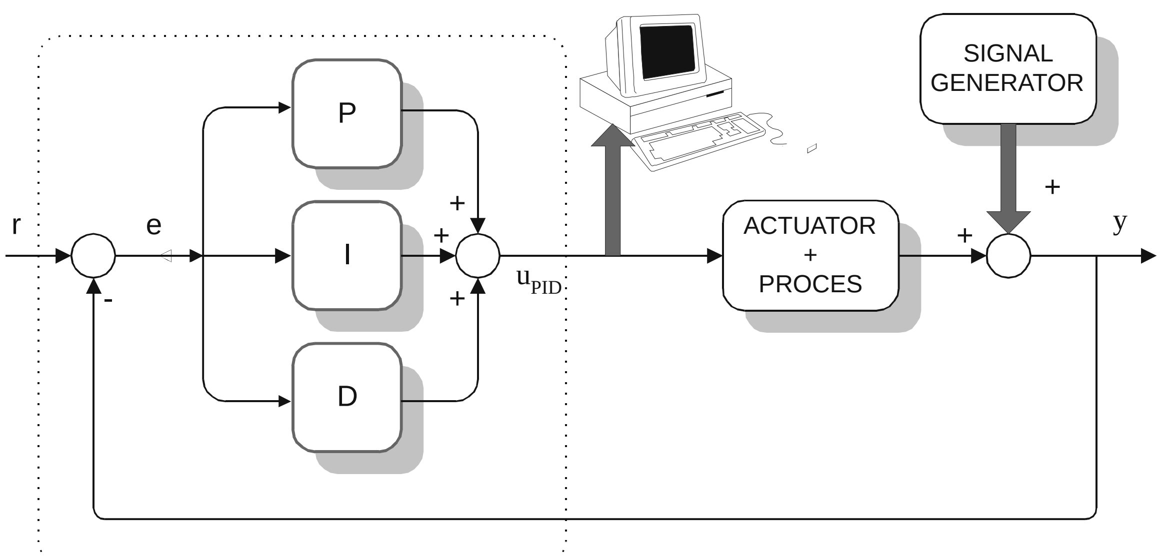


Fig. 1-24: Controller parameters identification – experiment setup

1.4.1 Identification of the controller gain K

Proportional gain of the controller can be measured form transfer function of proportional channel of the controller. In order to that one should disconnect derivative and integral channel, or, if that is not possible one should set T_d on zero and T_i on very large value. In that way the PID controller is reduced to P controller only. From the response (Fig. 1-25) of such P controller it is possible to compute the proportional gain of the controller:

$$K = \frac{\Delta u}{\Delta y}. \qquad (1\text{-}36)$$

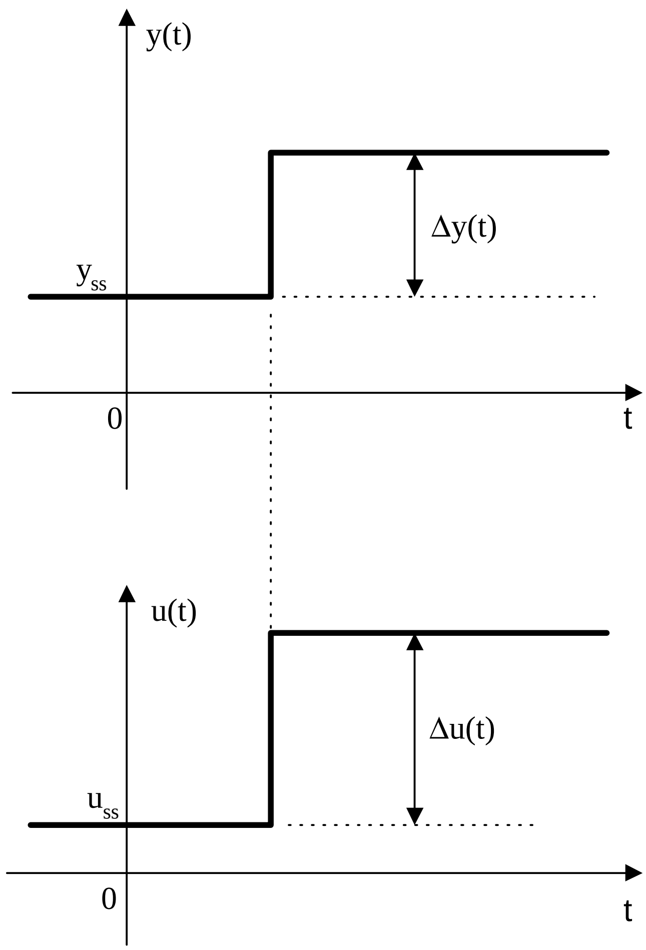


Fig. 1-25: P controller response on step input

1.4.2 Identification of PID controller integral time constant T_i

Integral time constant oif the controller can be determined from the PI controller response (so, T_d is set to zero or derivative channel is disconnected). From the response given in (Fig. 1-26) the integral time constant is determined as:

$$T_i = K \frac{\Delta y}{\Delta u} \Delta t \,. \qquad (1\text{-}37)$$

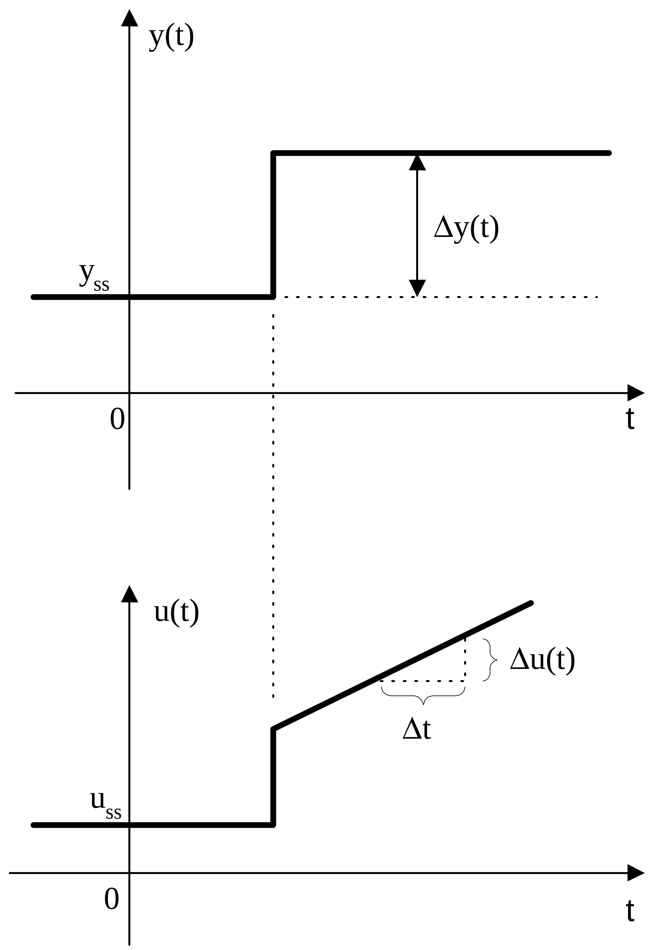


Fig. 1-26: PI controller response on step input

1.4.3 Identification of PID controller derivative time constant T_d

Derivative time constant of the controller is determined from PD controller response on ramp input. Integral channel is disconnected or time constant T_i is set to very large value ($T_i \to \infty$). If responses of P and PD controller on ramp signal e(t) = At are compared, it can be seen that response for P controller is given as:

$$u_p(t) = Ke(t) = KAt\,, \tag{1-38}$$

while the response for PD controller (with $u_0 = 0$) is given as:

$$u_{PD}(t) = Ke(t) + KT_d \frac{de(t)}{dt} = KA(t + T_d)\,, \tag{1-39}$$

The only difference in response is that PD controller gives value of control signal T_d seconds before P controller. From the response on the ramp input (Fig. 1-27) it is possible then to measure time constant T_d.

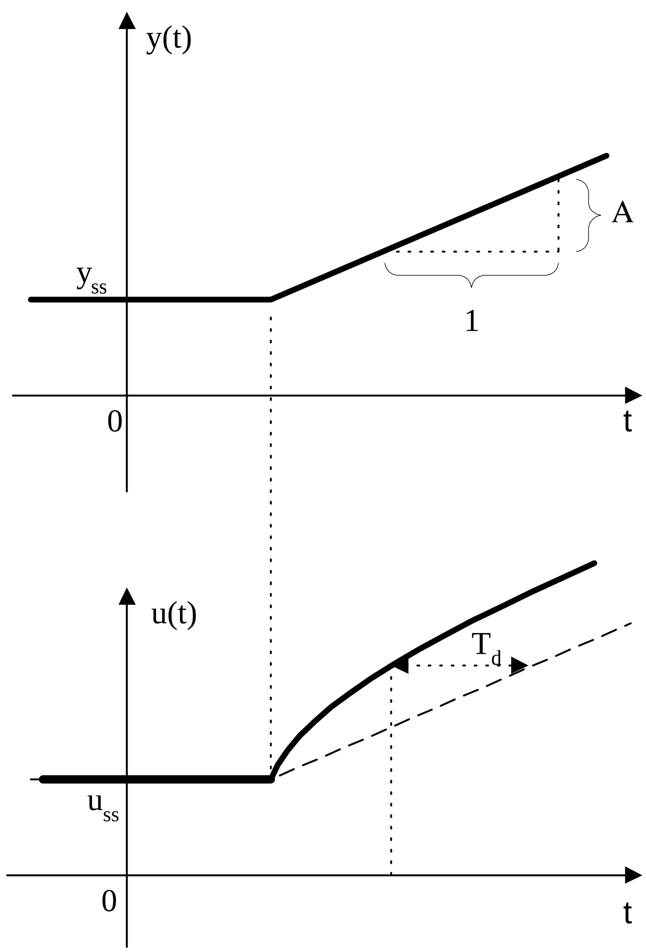


Fig. 1-27: PD controller response on ramp input

1.4.4 Identification of PID controller topology

In order to identify controller topology it should work with all modes active. Then from the step response of PID controller (Fig. 1-28) can be determined its topology.

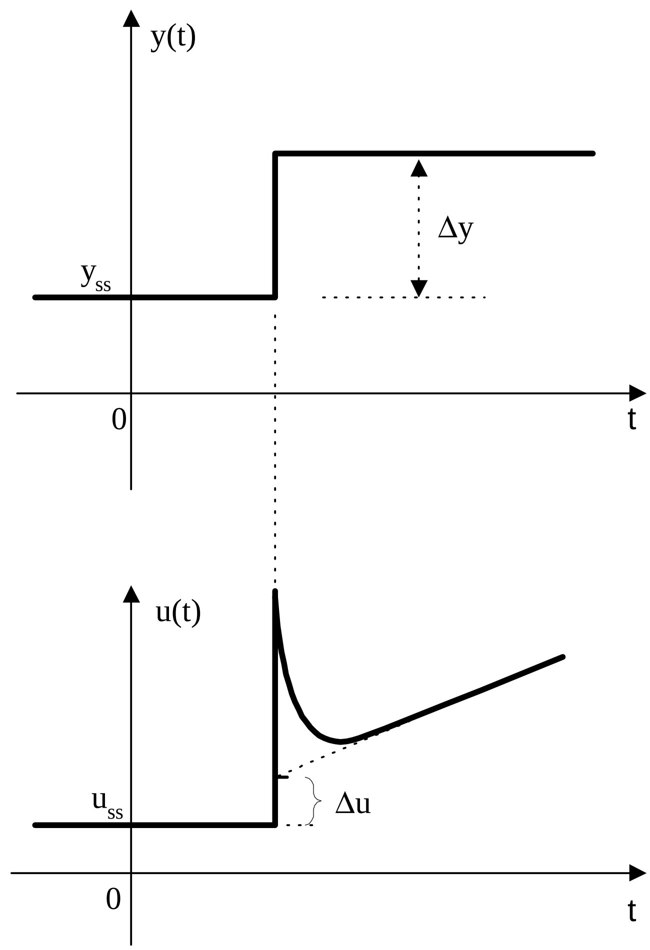


Fig. 1-28: Step response of PID controller

The structure of the controller can be determined from ratio $\frac{\Delta u}{\Delta y}$.

If:

- $\frac{\Delta u}{\Delta y} = K$ then structure of the controller is parallel
- $\frac{\Delta u}{\Delta y} = K(1+\frac{T_d}{T_i})$ then structure of the controller is serial

For example, if PID parameters are $K = T_d = T_i = 1$ and if $\Delta y = 1$, then for parallel structure $\Delta u = 1$ and for serial structure $\Delta u = 2$.

1.5 Experimental tuning of PID controller parameters

There are several recommendations for tuning PID controller parameters and for experimental determination of process characteristics to obtain process variables which will be used to set controller parameters. These procedures can be applied when mathematical model of the process is known and also when it is unknown. In any case, these recommendations can be used for initial tuning of the controller and then user can perform fine tuning using more detail knowledge of the process. The most often used recommendations are Ziegler-Nichols, Cohen-Coon and Chien-Hrones-Reswick procedures.

1.5.1 Parameter tuning according to Ziegler-Nichols recommendations

Ziegler-Nichols tuning ([1]) is used for P, PI and PID controllers. It has to be noted that controllers tuned using this procedure are tuned for control, not tracking. Thus, controllers with parameters tuned according to Ziegler-Nichols recommendation will perform well in disturbance rejection, but it will perform poor in tracking reference changes. There are two experiments used to obtain process variables needed to determine control parameters from the tables:

- open loop experiment – recording process transfer function
- closed loop experiment – leading closed loop system into self oscillations or to stability margin

1.5.1.1 Open loop experiment

Many industrial processes are stable with monotonous transfer function with transport delay. These processes can be described using the following transfer function:

$$G_p(s) = \frac{K_p}{1+sT_p} e^{-\tau s}, \qquad (1\text{-}40)$$

In Fig. 1-29 is shown step response of the system on step input with amplitude A. From this transfer 3 parameters can be obtained:

- static process gain $K_p = \frac{\Delta y}{\Delta u}$
- process transport delay τ
- process time constant T_p

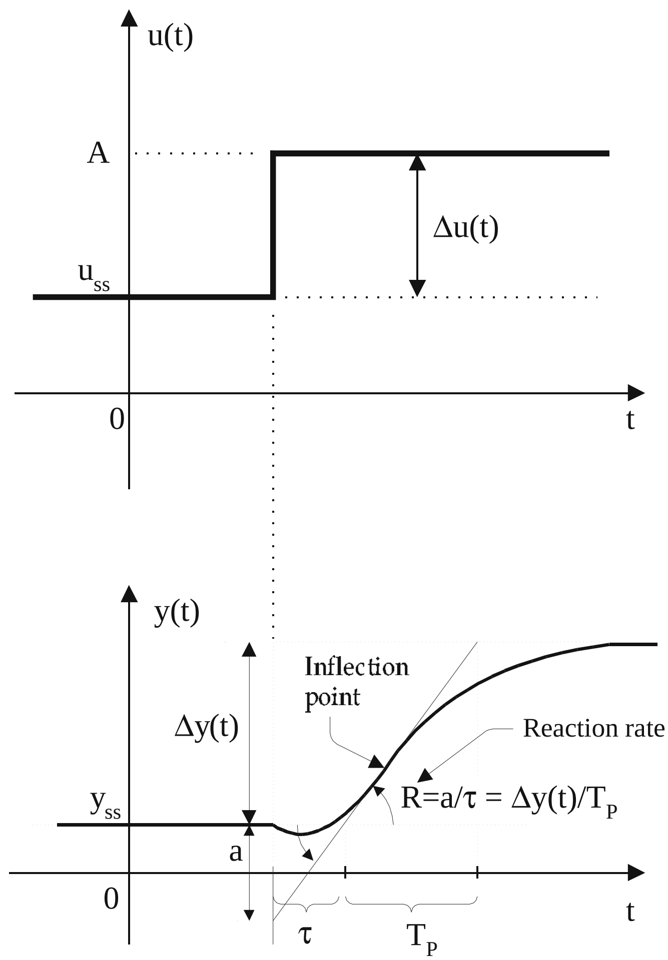


Fig. 1-29: Step response of the process

From these parameters a constant a is computed:

$$a = \mu K_p ;\text{where } \mu = \frac{\tau}{T_p}, \qquad (1\text{-}41)$$

This experiment cannot be performed if:

- transfer function is not monotonous
- process has astatical mode of 1^{st} or higher order
- if process is unstable

Ziegler-Nichols controller parameters recommendations are given in Table 2.

Ziegler-Nichols recommendations – open loop experiment			
Controller type	K	T_i	T_d
P	1/a	N/A	N/A
PI	0.9/a	3τ	N/A
PID parallel	1.2/a	2τ	$\tau/2$
PID serial	0.6/a	τ	τ

Table 2: Ziegler-Nichols recommendation – open loop experiment

Controller parameters given in Table 2 are obtained after simulations and experiments on a large number of processes. They are based on quarter amplitude damping requirement (amplitude of the response has to be one quarter of the amplitude of the previous cycle in the response). Criterion used by Ziegler and Nichols to tune parameters is actually IAE (Integral of absolute error – IAE) criterion that is mathematically described as:

$$J_{IAE} = \int_0^\infty |e(t)| dt = \int_0^\infty |r(t) - y(t)| dt \text{ with } r(t) = 1, \tag{1-42}$$

A second order system with quarter decay ratio (damping factor) $\zeta = 0.21$ if there are no finite zeros. Shinskey compares this behavior with the system with amplitude margin 2 (6 dB). Although this is not completely correct, it still gives a useful approximation for the system controlled with the controller with parameters tuned using Ziegler-Nichols recommendations. Because of the chosen damping ($\zeta = 0.21$), a shortcoming of the systems controlled with the controller parameters tuned as described above is weak damping. That will result in oscillatory dynamics of the closed loop system when reference is changed. It is possible to design systems with better damping by adjusting expressions in Table 2.

Ziegler-Nichols recommendations should be used for systems with $0.1 < \frac{\tau}{T_p} < 1$. For a larger values $\frac{\tau}{T_p}$ it is better to use control laws that can compensate for transport delay: Otto-Smith predictor, PIP controller, IMC controller or others. Also, Cohen-Coon recommendations will give better results in such cases. For smaller values of $\frac{\tau}{T_p}$ better performance can achieved using higher order compensators.

Transfer function recording experiment is not always easy to automate. It is difficult to know a priori what amplitude A of the step signal should be used or to determine when steady state is achieved. Step reference change should be large enough for the step response of the system to be distinguishable from the noise, but not too large in order not to disturb process itself (if experiment is conducted "on-line", during normal manufacturing process). Disturbances will also have impact on the experiment result.

If it is not possible to conduct open loop experiment, then one should try closed loop experiment.

1.5.1.2 Closed loop experiment

With this procedure no process model is assumed. Procedure is based on measurements only. Experiment can be conducted with stable and unstable processes. System is tested in closed loop with P controller (integral and derivative mode are disconnected). P controller gain is increased until system reaches stability margin (oscillations). Block diagram of experiment setup is shown in Fig. 1-30.

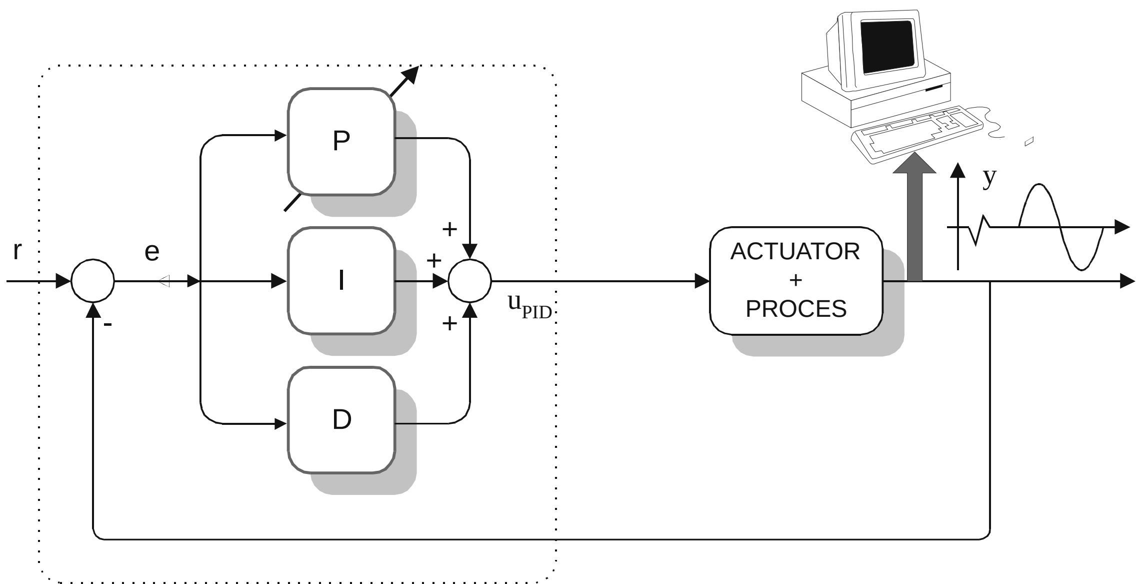


Fig. 1-30: Ziegler-Nichols closed loop experiment setup

When oscillations with constant amplitude and period are established, it is possible to determine oscillations period (ultimate period) T_u and controller (critical) gain (ultimate gain) with which oscillations where established. During experiment, by changing reference, it is possible to determine process static gain K_p as ratio between response and reference changes at steady state.

$$K_p = \frac{\Delta y}{\Delta u}. \qquad (1\text{-}43)$$

Based on experimentally obtained T_u and K_u Ziegler and Nichols have given the following table for controller parameters (assuming quarter decay ratio criterion):

Ziegler-Nichols recommendations – closed loop experiment			
Controller type	K	T_i	T_d
P	0.5 K_u	N/A	N/A
PI	0.45 K_u	0.833 T_u	N/A
PID parallel	0.6 K_u	0.5 T_u	0.125 T_u
PID serial	0.6 K_u	$6/T_u$	$1/T_u$

Table 3: Ziegler-Nichols recommendation – closed loop experiment

Gain product $\chi = K_pK_u$ can be treated as maximal (critical) gain of the open loop circuit and used to determine should Ziegler-Nichols tuned parameters be applied.

It is usually recommended that Ziegler-Nichols be used for parameter tuning if

$$2 < K_pK_u < 20. \qquad (1\text{-}44)$$

Also:

1. If $\chi = K_pK_u < 2$, then control laws that can compensate for transport delay should be used.
2. If $\chi > 20$, then better results can be achieved by more complex control algorithms
3. If $1.5 < \chi < 2$, then PID controller can be used if requirements on the control system performance are not very strict. Ziegler-Nichols procedures have to be modified to achieve good performance. Other structures should be tried (Otto-Smith predictor, Imc etc.).
4. If $\chi < 1.5$, then PI controller can be tried if requirements on the control system performance are not very strict. Derivative mode will not be of significant use. Other structures can be also recommended.

The gain of the open loop with P controller tuned according to Ziegler-Nichols recommendations (open loop experiment) is

$$\frac{K_p}{a} = \frac{T_p}{\tau} = \frac{1}{\mu} \approx \frac{\chi}{2}. \qquad (1\text{-}45)$$

Open loop gain is approximately $\frac{\chi}{2}$. If requirements on accuracy of the system in steady state are known then it is possible to determine if the requirements will be meet by using only P controller or I mode should be also used.

With Ziegler-Nichols rules, the largest error on unit step process reference change is given approximately with:

- For P controller (tuned according Ziegler-Nichols recommendations):

$$e_{max} \approx \frac{0.4}{K};\ t_{max} \approx \frac{T_i}{2}, \tag{1-46}$$

- For PI controller (tuned according Ziegler-Nichols recommendations):

$$e_{max} \approx \frac{0.6}{K};\ t_{max} \approx T_i. \tag{1-47}$$

The rise time of the closed loop with controller tuned according Ziegler-Nichols recommendations will be approximately equal to transport delay $t_r \approx \tau$.

1.6 Parameter tuning according to Cohen-Coon recommendations

Cohen-Coon tuning procedure uses the parameters obtained from open loop transfer function experiment (section 1.5.1.1 Open loop experiment). Cohen-Coon recommendations are given in Table 4.

Cohen-Coon recommendations			
Controller type	K	T_i	T_d
P	$\frac{1}{K_p}(0.35+\frac{1}{\mu})$	N/A	N/A
PI	$\frac{1}{K_p}(0.083+\frac{0.9}{\mu})$	$\frac{3.3+0.31\mu}{1+2.2\mu}\tau$	N/A
PD	$\frac{1}{K_p}(0.16+\frac{1.24}{\mu})$	N/A	$\frac{0.27-0.088\mu}{1+0.13\mu}\tau$
PID parallel	$\frac{1}{K_p}(0.25+\frac{1.35}{\mu})$	$\frac{2.5+0.46\mu}{1+0.61\mu}\tau$	$\frac{0.37}{1+0.19\mu}u$

Table 4: Cohen-Coon recommendations

The criterion used here is the same as with Ziegler-Nichols method (Quarter amplitude damping.). When transport delay is small compared with process time constant (small μ), Ziegler-Nichols and Cohen-Coon method will give similar controller parameters. However, when transport delay is large (large μ) Cohen-Coon method is recommended since according to Cohen-Coon method derivative should tend toward 0 for PID controller. That is more appropriate because for big delays (large τ) derivative mode should not be used.

1.7 Parameter tuning according to Chien-Hrones-Reswick (CHR) recommendations

In process industry controller parameters are often tuned according to CHR recommendations. They are based on time parameters of open loop step reference change response (Fig. 1-29). Chien, Hrones and Reswick ([4]) gave also recommendation for the choice of the type of the controller. Controller type is chosen, according to parameter R, from Table 5.

CHR recommendations for choice of controller type	
Controller type	$R = \frac{T_p}{\tau} = \frac{1}{\mu}$
P	$R > 10$
PI	$7.5 < R < 10$
PID parallel	$3 < R < 7.5$
Higher order	$R < 3$

Table 5: CHR recommendations for choice of controller type

CHR recommendations are given for two cases:

- transfer characteristic of closed loop should be aperiodic
- transfer characteristic of closed loop should be oscillatory with 20% overshoot

For higher order processes only approximately similar performance can be achieved.

For aperiodic response controller parameters have to be tuned according to Table 6

CHR recommendations for aperiodic response			
Controller type	K	T_i	T_d
P	$0.3\ R/K_p$	N/A	N/A
PI	$0.35\ R/K_p$	$1.2\ T_p$	N/A
PD	$0.6\ R/K_p$	T_p	$0.5\ \tau$

Table 6: CHR recommendations for aperiodic response

For oscillatory response with 20% overshoot controller parameters should be tuned according to Table 7.

CHR recommendations for aperiodic oscillatory response with 20% overshoot			
Controller type	K	T_i	T_d
P	0.7 R/K_p	N/A	N/A
PI	0.6 R/K_p	T_p	N/A
PD	0.95 R/K_p	1.35 T_p	0.47 τ

Table 7: CHR recommendations for oscillatory response with 20% overshoot

2. TABLE OF FIGURES

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4. REFERENCES

[1] Ziegler, J.G, Nichols, N.B., "Optimum settings for automatic controllers", Trans.ASME, vol. 64, pp. 759-768, 1942

[2] Cohen, G.H., Coon, G.A. "Theoretical consideration of retarded control", Trans. ASME vol. 75, pp.827-834, 1953

[3] Netushil at al. *Theory of Automatic Control*, Mir, Moscow 1978.

[4] Chien, K.L., Hrones, J.A., Reswick, J.B., "On the automatic control of generalized passive systems", Trans. ASME, 74, 175, 1952

[5] Åström, K.J., Hägglund, T., *PID Control – Theory, Design and Tuning*, Instrument Society of America, Research Triangle Park, NC, 2nd ed., 1995

[6] Levine, W.S. ed., *The control handbook*, CRC Press, 1995